University of South Carolina

MATH 544-001

Practice Midterm Examination 3 B

April 4, 2024

Closed book examination Time: 75 minutes

Instructions:

Notes, books, computer, phones, calculators or other aids are **not** allowed. Please write on only one side of each page. If you need more space than is provided, then ask for extra paper from the proctor. Simplify your final answers. Full credit will not be awarded for insufficient accompanying work.

There are 30 total points available.

Problem 1. (5 points) Find a basis for the null space of the following matrix:

$$\begin{pmatrix} 2 & 0 & 4 & 6 & -2 \\ 4 & 0 & 8 & 12 & -4 \\ 1 & 1 & 3 & 4 & 1 \\ 0 & 0 & 0 & 2 & 3 \end{pmatrix}$$

Problem 2. (5 points) Determine whether the set

$$\mathcal{B} = \left\{ \begin{pmatrix} 1 \\ 4 \\ 3 \end{pmatrix}, \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 5 \\ 6 \end{pmatrix} \right\}$$

is a basis for $\mathbb{R}^3$.

Problem 3. (5 points) Compute the change-of-basis matrix $P_{\mathcal{C}\to\mathcal{B}}$ where

$$\mathcal{B} = \left\{ \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right\} \text{ and } \mathcal{C} = \left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 4 \\ 1 \end{pmatrix} \right\}$$

are bases of $\mathbb{R}^3$.

Determine whether each of the following statements are true or false. No justification is necessary.

Problem 4. (1 point) $\mathbb{R}^2$ is a subspace of $\mathbb{R}^3$.

Problem 5. (1 point) Let A be an $m \times n$ matrix. If the equation $A\mathbf{x} = \mathbf{b}$ has a solution for every $\mathbf{b}$, then $\mathrm{Col}(A) = \mathbb{R}^m$.

Problem 6. (1 point) If $H = \mathrm{Span}\{\mathbf{b}_1, \ldots, \mathbf{b}_p\}$, then $\mathbf{b}_1, \ldots, \mathbf{b}_p$ is a basis for H.

Problem 7. (1 point) The vector space $\mathbb{P}_3$ of polynomials of degree at most 3 is isomorphic to $\mathbb{R}^4$.

Problem 8. (1 point) A square matrix A is invertible if and only if $\mathrm{Nul}(A) = \{0\}$.

Problem 9. (5 points) Let $M_{m,n}$ be the vector space of $m \times n$ matrices. Let $f : M_{m,n} \to M_{n,m}$ be the function which takes a matrix A to its transpose A^T. Prove that f is a linear transformation.

Problem 10. (5 points) Let $T : V \to V$ be a linear transformation from a vector space V to itself. Prove that $\ker(T) \subseteq \ker(T \circ T)$.

The End