University of South Carolina

MATH 544-001

Practice Midterm Examination 3 A

April 4, 2024

Closed book examination Time: 75 minutes

Instructions:

Notes, books, computer, phones, calculators or other aids are **not** allowed. Please write on only one side of each page. If you need more space than is provided, then ask for extra paper from the proctor. Simplify your final answers. Full credit will not be awarded for insufficient accompanying work.

There are 30 total points available.

Problem 1. (5 points) Let $\mathbb{P}_3$ be the vector space of polynomials of degree less than or equal to 3 and let V be the subspace of $\mathbb{P}_3$ consisting of polynomials f such that $f(2) = 0$. Find a basis for V.

Problem 2. (5 points) Find a basis for the column space of the following matrix:

$$\begin{pmatrix} 1 & 2 & 1 & 0 \\ 0 & 0 & 6 & 2 \\ 4 & 8 & 4 & 0 \\ 2 & 4 & 5 & 1 \\ 0 & 0 & 3 & 1 \end{pmatrix}$$

Problem 3. (5 points) The set

$$\mathcal{B} = \left\{ \begin{pmatrix} 1 \\ 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ 4 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 4 \\ 1 \\ 4 \end{pmatrix}, \begin{pmatrix} 0 \\ 6 \\ 1 \\ 6 \end{pmatrix} \right\}$$

is a basis for $\mathbb{R}^3$. Find the coordinate vector $[\mathbf{x}]_{\mathcal{B}}$ where

$$\mathbf{x} = \begin{pmatrix} 4 \\ 2 \\ 0 \\ 5 \end{pmatrix}.$$

Determine whether each of the following statements are true or false. No justification is necessary.

Problem 4. (1 point) The null space of an $m \times n$ matrix is $\mathbb{R}^n$.

Problem 5. (1 point) Let A be an $m \times n$ matrix. If the equation $A\mathbf{x} = 0$ has a solution, then $\text{Nul}(A) = \mathbb{R}^n$.

Problem 6. (1 point) The image of a linear transformation is a vector space.

Problem 7. (1 point) The vector space of 3×4 matrices has dimension 12.

Problem 8. (1 point) The null space of a matrix has a unique basis.

Problem 9. (5 points) Let M_n be the vector space of square $n \times n$ matrices. A matrix $A \in M_n$ is antisymmetric if $A^T = -A$. Prove that the subset of antisymmetric matrices in M_n is a subspace.

Problem 10. (5 points) Let V be an n-dimensional vector space with basis $\mathcal{B}$. Prove that if $\mathbf{v}_1, \ldots, \mathbf{v}_n$ is a basis for V, then $[\mathbf{v}_1]_{\mathcal{B}}, \ldots, [\mathbf{v}_n]_{\mathcal{B}}$ is a basis for $\mathbb{R}^n$.

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