University of South Carolina
MATH 544-001

Practice Midterm Examination 2 B

February 29, 2024

Closed book examination   Time: 75 minutes

Instructions:

Notes, books, computer, phones, calculators or other aids are **not** allowed. Please write on only one side of each page. If you need more space than is provided, then ask for extra paper from the proctor. Simplify your final answers. Full credit will not be awarded for insufficient accompanying work.

There are 30 total points available.

Consider the matrices

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 1 \\ 2 & -1 \\ 1 & 1 \end{pmatrix}, \quad C = \begin{pmatrix} 4 & 1 \\ 7 & 2 \end{pmatrix}.$$

For each of the following, compute the matrix or indicate that the expression is undefined.

Problem 1. (2 points) BC^2

Problem 2. (3 points) $(AB)^T + 2C^{-1}$

Problem 3. (5 points) Find the inverse of the matrix

$$\begin{pmatrix} 1 & 2 & 1 & 0 & 2 \\ 0 & 1 & 0 & 0 & 5 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

or show that it does not exist.

Problem 4. (5 points) Find the determinant of the following matrix:

$$\begin{pmatrix} 4 & 1 & 1 \\ 2 & 1 & 5 \\ 2 & 9 & 1 \end{pmatrix}$$

Determine whether each of the following statements are true or false. No justification is necessary.

Problem 5. (1 point) Let A, B be $n \times n$ matrices. Then $A^T + B^T = (A + B)^T$

Problem 6. (1 point) If A is an $n \times n$ matrix, then equation $A\mathbf{x} = \mathbf{b}$ is consistent for every $\mathbf{b}$ in $\mathbb{R}^n$.

Problem 7. (1 point) Let A be an $n \times n$ matrix. If the columns of A span $\mathbb{R}^n$, then the columns of A are linearly independent.

Problem 8. (1 point) A row replacement operation does not affect the determinant of a matrix.

Problem 9. (1 point) Let A, B be $n \times n$ matrices. Then $\det(2A) = 2 \det(A)$.

Problem 10. (5 points) Let A be an $m \times n$ matrix, let B be an $n \times m$ matrix, and suppose $AB = I_m$. Prove that, for any $\mathbf{b}$ in $\mathbb{R}^m$, the equation $B\mathbf{x} = \mathbf{b}$ has a solution.

Problem 11. (5 points) Suppose A is an upper triangular $n \times n$ matrix. Prove that there exists a real number r such that $A + rI_n$ is <u>not</u> invertible.

The End