University of South Carolina

MATH 544-001

Midterm Examination 2

February 29, 2024

Closed book examination Time: 75 minutes

Instructions:

Notes, books, computer, phones, calculators or other aids are **not** allowed. Please write on only one side of each page. If you need more space than is provided, then ask for extra paper from the proctor. Simplify your final answers. Full credit will not be awarded for insufficient accompanying work.

There are 30 total points available.

Consider the matrices

$$A = \begin{pmatrix} 1 & 1 & 2 \\ 2 & 0 & 3 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & -2 \\ 2 & 1 \\ 3 & 0 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}.$$

For each of the following, compute the matrix or indicate that the expression is undefined.

Problem 1. (2 points) AA^T

Problem 2. (3 points) $C^{-1} + AB$

Problem 3. (5 points) Find the inverse of the matrix

$$\begin{pmatrix} 1 & 1 & 2 \\ 3 & -1 & 1 \\ 0 & 2 & 2 \end{pmatrix}$$

or show that it does not exist.

Problem 4. (5 points) Find the determinant of the following matrix:

$$\begin{pmatrix} 1 & 0 & 4 & 7 \\ 2 & 1 & 5 & 6 \\ 0 & 0 & 0 & 4 \\ 1 & 8 & 0 & 3 \end{pmatrix}$$

Determine whether each of the following statements are true or false. No justification is necessary.

Problem 5. (1 point) Let A be an $m \times n$ matrix and let B, C be $n \times p$ matrices. Then $AB + AC = A(B + C)$.

Problem 6. (1 point) If A is invertible, then the inverse of A^{-1} is A itself.

Problem 7. (1 point) Let A be an $n \times n$ matrix. If there is an $n \times n$ matrix D such that $AD = I_n$, then there is also an $n \times n$ matrix C such that $CA = I$.

Problem 8. (1 point) The determinant of a triangular matrix is the sum of the entries on the main diagonal.

Problem 9. (1 point) Let A, B be $n \times n$ matrices. Then $\det(A + B) = \det(A) + \det(B)$.

Problem 10. (5 points) Let A, B be $n \times n$ matrices. Prove that, if B is invertible, then $\det(A) = \det(B^{-1}AB)$.

Problem 11. (5 points) Let A, B be upper triangular $n \times n$ matrices. Prove that AB is also upper triangular.

The End