University of South Carolina
MATH 544-001

Practice Midterm Examination 1 B

February 1, 2024

Closed book examination Time: 75 minutes

Instructions:

Notes, books, computer, phones, calculators or other aids are **not** allowed. Please write on only one side of each page. If you need more space than is provided, then ask for extra paper from the proctor. Simplify your final answers. Full credit will not be awarded for insufficient accompanying work.

There are 30 total points available.

Problem 1. (5 points) Find the reduced row echelon form for the matrix

$$\begin{pmatrix} 1 & 3 & -2 & 4 \\ 1 & 3 & -2 & 4 \\ 2 & 3 & 1 & 0 \\ -2 & 3 & 0 & 4 \end{pmatrix}.$$

Problem 2. (5 points) Describe all solutions of the following linear system.

$$\begin{aligned} x_1 + 2x_3 - 4x_4 + 7x_6 &= 5 \\ x_2 + 3x_3 + 5x_4 + 6x_6 &= 2 \\ x_5 - 9x_6 &= 2 \end{aligned}$$

Problem 3. (5 points) Determine if the columns of the matrix

$$\begin{pmatrix} 3 & 9 & 3 & 7 \\ 1 & 3 & 1 & 5 \\ 1 & 3 & 2 & 0 \\ 2 & 6 & 0 & 4 \end{pmatrix}$$

form a linearly independent set.

Problem 4. (3 points) Determine whether $\mathbf{b}$ is in the image of the linear transformation $\mathbf{x} \mapsto A\mathbf{x}$ where

$$A = \begin{pmatrix} 1 & 4 & 8 \\ 0 & 2 & 4 \\ 0 & 0 & 1 \end{pmatrix} \text{ and } \mathbf{b} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}.$$

Problem 5. (2 points) Let $T : \mathbf{R}^2 \to \mathbf{R}^2$ by a linear transformation where $T(\mathbf{e}_1) = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ and $T(\mathbf{e}_2) = \begin{pmatrix} 5 \\ 1 \end{pmatrix}$. Find the standard matrix for T.

Determine whether each of the following statements are true or false. No justification is necessary.

Problem 6. (1 point) Every elementary row operation is reversible.

Problem 7. (1 point) When $\mathbf{u}$ and $\mathbf{v}$ are nonzero vectors, $\text{Span}\{\mathbf{u}, \mathbf{v}\}$ contains the line through $\mathbf{u}$ and the origin.

Problem 8. (1 point) Any finite linear combination of vectors can always be written in the form $A\mathbf{x}$ for a suitable matrix A and vector $\mathbf{x}$.

Problem 9. (1 point) A mapping $T : \mathbf{R}^n \to \mathbf{R}^m$ is injective if each vector in $\mathbf{R}^n$ maps onto a unique vector in $\mathbf{R}^m$.

Problem 10. (1 point) If a vector equation has a solution, then the zero vector is a solution

Problem 11. (5 points) Let S be a finite subset of $\mathbb{R}^n$. Prove that S is linearly independent if and only if every subset of S is linearly independent.

The End