University of South Carolina

MATH 544-001

Practice Midterm Examination 1 A

February 1, 2024

Closed book examination | Time: 75 minutes

Instructions:

Notes, books, computer, phones, calculators or other aids are **not** allowed. Please write on only one side of each page. If you need more space than is provided, then ask for extra paper from the proctor. Simplify your final answers. Full credit will not be awarded for insufficient accompanying work.

There are 30 total points available.

Problem 1. (5 points) Find the reduced row echelon form for the matrix

$$\begin{pmatrix} 1 & 1 & 6 & 2 & 9 & 0 \\ 2 & 2 & 2 & 0 & 1 & 0 \\ -3 & -3 & 4 & 2 & 1 & 0 \end{pmatrix}.$$

Problem 2. (5 points) Describe all solutions of $A\mathbf{x} = \mathbf{b}$ in parametric vector form, where

$$A = \begin{pmatrix} 1 & 9 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 1 & -4 \end{pmatrix} \text{ and } \mathbf{b} = \begin{pmatrix} 8 \\ 3 \\ 7 \end{pmatrix}.$$

Problem 3. (5 points) Determine if the vectors

$$\begin{pmatrix}1\\4\\6\end{pmatrix}, \begin{pmatrix}5\\3\\1\end{pmatrix}, \text{ and } \begin{pmatrix}2\\4\\0\end{pmatrix}.$$

are linearly independent.

Problem 4. (2 points) Let $A = \begin{pmatrix} 3 & -2 \\ 4 & 5 \end{pmatrix}$ and define $T : \mathbb{R}^2 \to \mathbb{R}^2$ by $T(\mathbf{x}) = A\mathbf{x}$. Find the image under T of $\mathbf{u} = \begin{pmatrix} -5 \\ 6 \end{pmatrix}$.

Problem 5. (3 points) Let $T : \mathbf{R}^2 \to \mathbf{R}^2$ be a linear transformation where $T(\mathbf{e}_1) = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$ and $T(\mathbf{e}_2) = \begin{pmatrix} 7 \\ 3 \end{pmatrix}$. Find $\mathbf{x}$ such that $T(\mathbf{x}) = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$.

Determine whether each of the following statements are true or false. No justification is necessary.

Problem 6. (1 point) A system of linear equations never has exactly 6 distinct solutions.

Problem 7. (1 point) Two matrices are row equivalent if the have the same number of rows.

Problem 8. (1 point) An example of a linear combination of the vectors $\mathbf{u}$ and $\mathbf{v}$ is the vector $\frac{1}{2}\mathbf{v}$.

Problem 9. (1 point) The equation $A\mathbf{x} = \mathbf{b}$ is consistent if the augmented matrix $[A\mathbf{b}]$ has a pivot position in every row.

Problem 10. (1 point) If S is a linearly dependent set, then each vector is a linear combination of the other vectors in S.

Problem 11. (5 points) Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation. Prove that if T maps a linearly independent set onto a linearly dependent set, then $T(\mathbf{x}) = \mathbf{0}$ has a nontrivial solution.

The End