University of South Carolina
MATH 544-001

Midterm Examination 1

February 1, 2024

Closed book examination Time: 75 minutes

Instructions:

Notes, books, computer, phones, calculators or other aids are **not** allowed. Please write on only one side of each page. If you need more space than is provided, then ask for extra paper from the proctor. Simplify your final answers. Full credit will not be awarded for insufficient accompanying work.

There are 30 total points available.

Problem 1. (5 points) Find the reduced row echelon form for the matrix

$$\begin{pmatrix} 2 & 1 & 6 & 2 \\ 4 & 1 & 0 & 2 \\ 3 & 1 & 2 & 0 \end{pmatrix}.$$

Problem 2. (5 points) Describe all solutions of $A\mathbf{x} = \mathbf{b}$ in parametric vector form, where

$$A = \begin{pmatrix} 1 & 0 & 2 & 0 & 3 \\ 0 & 1 & 4 & 0 & 5 \\ 0 & 0 & 0 & 1 & 6 \end{pmatrix} \text{ and } \mathbf{b} = \begin{pmatrix} 7 \\ 8 \\ 9 \end{pmatrix}.$$

Problem 3. (5 points) Determine if the vectors

$$\begin{pmatrix} 5 \\ -2 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}, \text{ and } \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}.$$

are linearly independent.

Problem 4. (2 points) Let $A = \begin{pmatrix} 2 & 3 \\ 4 & 1 \end{pmatrix}$ and define $T : \mathbb{R}^2 \to \mathbb{R}^2$ by $T(\mathbf{x}) = A\mathbf{x}$. Find the image under T of $\mathbf{u} = \begin{pmatrix} 5 \\ -2 \end{pmatrix}$.

Problem 5. (3 points) Let $T : \mathbf{R}^2 \to \mathbf{R}^2$ be a linear transformation where $T(\mathbf{e}_1) = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ and $T(\mathbf{e}_2) = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$. Find $\mathbf{x}$ such that $T(\mathbf{x}) = \begin{pmatrix} 7 \\ 3 \end{pmatrix}$.

Determine whether each of the following statements are true or false. No justification is necessary.

Problem 6. (1 point) An inconsistent system has more than one solution.

Problem 7. (1 point) The echelon form of a matrix is unique.

Problem 8. (1 point) If A is an $m \times n$ matrix whose columns do not span $\mathbb{R}^m$, then the equation $A\mathbf{x} = \mathbf{b}$ is inconsistent for some $\mathbf{b}$ in $\mathbb{R}^n$.

Problem 9. (1 point) If $\mathbf{x}$ is a nontrivial solution of $A\mathbf{x} = \mathbf{0}$, then every entry in $\mathbf{x}$ is nonzero.

Problem 10. (1 point) Any set containing the zero vector is linearly dependent.

Problem 11. (5 points) Suppose $\mathbf{v}_1, \ldots, \mathbf{v}_p$ span $\mathbb{R}^n$. Suppose $T : \mathbb{R}^n \to \mathbb{R}^m$ is a linear transformation such that $T(\mathbf{v}_i) = 0$ for all i from $1, \ldots, p$. Prove that $T(\mathbf{x}) = 0$ for every vector $\mathbf{x} \in \mathbb{R}^n$.

The End