

University of South Carolina

MATH 241-H01

Practice Midterm Examination 3 B

April 6, 2023

Closed book examination

Time: 75 minutes

Instructions:

Notes, books, computer, phones, calculators or other aids are **not** allowed. Please write on only one side of each page. If you need more space than is provided, then ask for extra paper from the proctor. Simplify your final answers. Full credit will not be awarded for insufficient accompanying work.

There are 55 points available, but the exam is **out of** 50.
(In other words, there are 5 bonus points available)

Problem 1. (10 points) Find the minimum and maximum distances between the ellipse $x^2 + xy + 2y^2 = 1$ and the origin.

Problem 2. (4 points) Find $\int_{-1}^1 \int_0^1 \int_0^2 xy + yz \, dx \, dy \, dz$.

Problem 3. (4 points) Find $\int_0^2 \int_{y^2}^4 2xy - y^2 \, dx \, dy$.

Problem 4. (4 points) Change the order of integration in

$$\int_0^1 \int_{x-1}^{1-x} f(x, y) \, dy \, dx.$$

(This may involve breaking the integral up into multiple integrals.)

Problem 5. (5 points) Find $\iint_R r \, dA$ where R is the region bounded by $r = \sin(2\theta)$ in the first quadrant where $x \geq 0$ and $y \geq 0$.

Problem 6. (4 points) Change the order of integration of x and y in

$$\int_0^1 \int_0^z \int_0^y f(x, y, z) \, dx \, dy \, dz.$$

(This may involve breaking the integral up into multiple integrals.)

Problem 7. (4 points) Rewrite the following integral using spherical coordinates

$$\int_0^4 \int_0^{\sqrt{16-x^2}} \int_{-\sqrt{16-x^2-y^2}}^{\sqrt{16-x^2-y^2}} (x^2 + y^2 + z^2)^2 \, dz \, dy \, dx.$$

(Do not evaluate the integral.)

Problem 8. (10 points) Find the volume of the solid bounded by $z = 12 - 3x - y$ in the first octant.

Problem 9. (10 points) Find $\iiint_E z \, dV$ where

$$E = \{(x, y, z) \mid x^2 + y^2 + z^2 - 2z \leq 0, \sqrt{x^2 + y^2} \leq z\}.$$

The End