University of South Carolina
MATH 241-H01

Practice Midterm Examination 3 A

April 6, 2023

Closed book examination Time: 75 minutes

Instructions:

Notes, books, computer, phones, calculators or other aids are **not** allowed. Please write on only one side of each page. If you need more space than is provided, then ask for extra paper from the proctor. Simplify your final answers. Full credit will not be awarded for insufficient accompanying work.

There are 55 points available, but the exam is **out of** 50.
(In other words, there are 5 bonus points available)

Problem 1. (10 points) Minimize $f(x, y, z) = x^2 + y^2 + z^2$ subject to the constraints $x + y + z = 9$ and $x + 2y + 3z = 20$.

Problem 2. (4 points) Find $\int_0^2 \int_0^1 \int_0^1 (x+y)^2 z \; dz \; dy \; dx$.

Problem 3. (4 points) Find $\int_0^1 \int_{x^2}^{2-x^2} y + x \; dy \; dx$.

Problem 4. (4 points) Change the order of integration in

$$\int_{-1}^{1}\int_{x^2-1}^{1} f(x,y)\ dy\ dx.$$

(This may involve breaking the integral up into multiple integrals.)

Problem 5. (5 points) Find $\iint_R x\ dA$ where R is the intersection of the circle $r \leq 2$ and the half-plane $x \geq 0$.

Problem 6. (4 points) Write $\iiint_E f(x,y,z)\ dV$ as an iterated integral in cylindrical coordinates, where E is the solid located inside the cylinder $x^2+y^2=1$ and between the circular paraboloids $z=3-x^2-y^2$ and $z=x^2+y^2$. (Do not evaluate the integral.)

Problem 7. (4 points) Rewrite the following integral using cylindrical coordinates

$$\int_0^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} \int_{x^2+y^2}^{\sqrt{x^2+y^2}} xz\ dz\ dx\ dy.$$

(Do not evaluate the integral.)

Problem 8. (10 points) Find the volume of the solid bounded by the planes $x + y = 1$, $x - y = 1$, $x = 0$, $z = 0$ and $z = 10$.

Problem 9. (10 points) Find $\iiint_E \sqrt{x^2+y^2}\, dV$ where E is bounded above the half-sphere $x^2+y^2+z^2 = 9$ with $z \geq 0$ and below by the cone $2z^2 = x^2 + y^2$.

The End