# University of South Carolina

# MATH 241-H01

Midterm Examination 3

April 6, 2023

Closed book examination Time: 75 minutes

**Instructions:**

Notes, books, computer, phones, calculators or other aids are **not** allowed. Please write on only one side of each page. If you need more space than is provided, then ask for extra paper from the proctor. Simplify your final answers. Full credit will not be awarded for insufficient accompanying work.

There are 55 points available, but the exam is **out of** 50.
(In other words, there are 5 bonus points available)

**Problem 1.** (10 points) Find the maximum and minimum values of $f(x, y) = x^2 + y^2$ subject to the constraint $x^2 + y^2 = 2x + 4y$.

**Problem 2.** (4 points) Find $\int_0^1 \int_0^2 \int_0^3 x^2y + z \; dy \; dx \; dz$.

**Problem 3.** (4 points) Find $\int_0^1 \int_y^{y^2} x + 2 \; dx \; dy$.

**Problem 4.** (4 points) Change the order of integration in

$$\int_0^2 \int_{x^2}^4 f(x,y)\ dy\ dx.$$

(This may involve breaking the integral up into multiple integrals.)

**Problem 5.** (5 points) Find $\iint_R x\ dA$ where $R$ is the semicircular region given by the inequalities $x^2 + y^2 \leq 3$ and $x \geq 0$.

**Problem 6.** (4 points) Change the order of integration of $y$ and $z$ in

$$\int_1^2 \int_0^x \int_0^{2x-2z} f(x,y,z)\ dy\ dz\ dx$$

(This may involve breaking the integral up into multiple integrals.)

**Problem 7.** (4 points) Rewrite the following integral using spherical coordinates

$$\int_0^{2\pi} \int_0^{\sqrt{2}} \int_r^{\sqrt{4-r^2}} r\ dz\ dr\ d\theta.$$

(Do not evaluate the integral.)

**Problem 8.** (10 points) Find the volume of the tetrahedron in the first octant bounded by the coordinate planes and the plane passing through the points $(1, 0, 0)$, $(0, 2, 0)$ and $(0, 0, 1)$.

**Problem 9.** (10 points) Find $\iiint_E z \, dV$ where $E$ is the solid region below by the cone $z = \sqrt{x^2 + y^2}$ and above by the paraboloid $z = 2 - x^2 - y^2$

**The End**