University of South Carolina
MATH 241-H01

Practice Midterm Examination 2 A

March 2, 2023

Closed book examination Time: 75 minutes

Instructions:

Notes, books, computer, phones, calculators or other aids are **not** allowed. Please write on only one side of each page. If you need more space than is provided, then ask for extra paper from the proctor. Simplify your final answers. Full credit will not be awarded for insufficient accompanying work.

There are 55 points available, but the exam is **out of** 50.
(In other words, there are 5 bonus points available)

Problem 1. (3 points) Describe the domain and range of the function

$$g(x, y) = \sqrt{1 - (x - 2)^2 - y^2}.$$

Problem 2. (5 points) Find and sketch the level curves $f(x, y) = c$ on the same set of axes for the function

$$f(x, y) = x^3 - y$$

at the values $c = -1, 0, 2$.

Problem 3. (3 points) Find the limit $\lim\limits_{(x,y)\to(1,1)} \dfrac{x^6 - y^3}{x^2 - y}$.

Problem 4. (3 points) Find the limit $\lim\limits_{(x,y)\to(0,2)} \dfrac{\ln(x^2 + y)}{x + y}$.

Problem 5. (3 points) Determine whether the function

$$f(x,y) = \begin{cases} \frac{x^2-y^2}{x^2+y^2+1} & \text{if } (x,y) \neq (0,0) \\ 1 & \text{if } (x,y) = (0,0) \end{cases}$$

is continuous at $(0,0)$.

Problem 6. (5 points) Let $z = \sin(x^2 - 2y)$. Find $\dfrac{\partial z}{\partial x}$, $\dfrac{\partial z}{\partial y}$, $\dfrac{\partial^2 z}{\partial x^2}$, $\dfrac{\partial^2 z}{\partial x \partial y}$, and $\dfrac{\partial^2 z}{\partial y^2}$.

Problem 7. (3 points) If $w = x^2 + y^2 - 3xy$, $x = s^2 - t$, and $y = t^2 - s$, find $\dfrac{\partial w}{\partial s}$ and $\dfrac{\partial w}{\partial t}$.

Problem 8. (5 points) Given $f(x, y) = e^{4-3x+y}$, approximate $f(2.1, 1.9)$ using a linear approximation at the point $(2, 2)$.

Problem 9. (5 points) Find the gradient of $f(x, y, z) = 4xz - e^{xy} + 3y^2z$ at the point $(0, 2, 4)$.

Problem 10. (10 points) For the function

$$f(x, y) = x^3 + y^3 - 300x - 75y - 3,$$

find all the critical points and use the second derivative test to determine, if possible, whether each is a maximum, minimum, or saddle point.

Problem 11. (10 points) Find the absolute extrema of the function

$$f(x, y) = x^2 + y^2 + 3x - 2$$

on the closed and bounded set given by region

$$R = \left\{(x, y) : x^2 + y^2 \leq 9\right\}.$$

The End