

University of South Carolina
MATH 241-H01
Midterm Examination 2
March 2, 2023

Closed book examination

Time: 75 minutes

Instructions:

Notes, books, computer, phones, calculators or other aids are **not** allowed. Please write on only one side of each page. If you need more space than is provided, then ask for extra paper from the proctor. Simplify your final answers. Full credit will not be awarded for insufficient accompanying work.

There are 55 points available, but the exam is **out of** 50.
(In other words, there are 5 bonus points available)

Problem 1. (3 points) Describe the domain and range of the function

$$f(x, y) = \frac{1}{x^2 + y^2}.$$

Problem 2. (5 points) Find and sketch the level curves $f(x, y) = c$ on the same set of axes for the function

$$f(x, y) = \frac{y}{x^2 + 1}$$

at the values $c = -1, 0, 1, 2$.

Problem 3. (3 points) Find the limit $\lim_{(x,y) \rightarrow (6,2)} \frac{e^{3y-x}}{x^2 - y}$.

Problem 4. (3 points) Find the limit $\lim_{(x,y) \rightarrow (2,2)} \frac{x^4 - y^4}{x^2 - y^2}$.

Problem 5. (3 points) Show the limit $\lim_{(x,y) \rightarrow (0,0)} \frac{xy - y^3}{x^2 + y^2}$ does not exist by considering the paths $y = -x$ and $y = x$.

Problem 6. (3 points) If $z = x^2 - 2xy + 3y$, $x = u^2 + v$, and $y = 2uv + 1$, find $\frac{\partial z}{\partial u}$.

Problem 7. (5 points) Find the equation for the tangent plane to the surface

$$z = 3x^2 + 4y^2 - 2xy + 3$$

at $(x, y) = (-1, 1)$.

Problem 8. (5 points) Given $f(x, y) = \sqrt{x^2 + y^2}$, approximate $f(4.1, 2.9)$ using a linear approximation at the point $(4, 3)$.

Problem 9. (5 points) Find the directional derivative of the function

$$f(x, y) = e^{x^2 - y^2}$$

at the point $(1, 2)$ in the direction of $\mathbf{u} = \left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle$.

Problem 10. (10 points) For the function

$$f(x, y) = x^3 - y^3 - 2xy + 7,$$

find all the critical points and use the second derivative test to determine, if possible, whether each is a maximum, minimum, or saddle point.

Problem 11. (10 points) Find the absolute extrema of the function

$$f(x, y) = 6xy - 4x^3 - 3y^2$$

on the closed and bounded rectangle given by $-1 \leq x \leq 1$ and $-1 \leq y \leq 1$.

The End