University of South Carolina
MATH 241-H01

Practice Midterm Examination 1 B

February 2, 2023

Closed book examination Time: 75 minutes

Instructions:

Notes, books, computer, phones, calculators or other aids are **not** allowed. Please write on only one side of each page. If you need more space than is provided, then ask for extra paper from the proctor. Simplify your final answers. Full credit will not be awarded for insufficient accompanying work.

There are 55 points available, but the exam is **out of** 50.
(In other words, there are 5 bonus points available)

Problem 1. (3 points) A vector $\mathbf{v}$ has initial point $(6, 6, 3)$ and terminal point $(2, -2, 2)$. Find the unit vector in the direction of $\mathbf{v}$. Express the answer in component form.

Problem 2. (3 points) Find the angle θ that the vector $\mathbf{u} = \langle 3\sqrt{2}, -3\sqrt{2} \rangle$ makes with the positive direction of the x-axis, in a counter-clockwise direction.

Problem 3. (3 points) Find a vector of magnitude 4 that points in the opposite direction of the vector $\overrightarrow{PQ}$ where $P = (3, 1)$ and $Q = (-1, 4)$.

Problem 4. (3 points) Given vectors $\mathbf{a} = \langle 2, 1, 1\rangle$ and $\mathbf{b} = \langle 2, -3, 4\rangle$, compute $\mathbf{a} \times \mathbf{b}$.

Problem 5. (3 points) Given vectors $\mathbf{a} = 2\mathbf{i} - \mathbf{k}$, $\mathbf{b} = 3\mathbf{i} + 2\mathbf{j}$, and $\mathbf{c} = \mathbf{j} - \mathbf{k}$, compute $(\mathbf{a} \cdot \mathbf{b})\mathbf{c}$.

Problem 6. (3 points) Find a vector $\mathbf{c}$ that is orthogonal to both $\mathbf{a} = \langle 3, -1, 1\rangle$ and $\mathbf{b} = \langle 1, 5, 0\rangle$.

Problem 7. (3 points) Determine the vector projection $\operatorname{proj}_{\mathbf{u}} \mathbf{v}$ of the vector $\mathbf{v} = \langle 2, 1 \rangle$ onto the vector $\mathbf{u} = \langle 3, 1 \rangle$.

Problem 8. (3 points) Find the volume of the parallelepiped with the adjacent edges $\mathbf{u} = \langle -3, 5, -1 \rangle$, $\mathbf{v} = \langle 0, 2, -2 \rangle$, and $\mathbf{w} = \langle 3, 1, 1 \rangle$.

Problem 9. (3 points) Let L be the line passing through the point $P = (0, -1, 1)$ with direction $\mathbf{v} = \langle 1, 1, 2 \rangle$. Find symmetric equations of the line L.

Problem 10. (3 points) Find the distance from the point $P = (1, 2, -1)$ to the plane of equation $x - 2y + z = 0$.

Problem 11. (3 points) Find the equation of the plane that passes through the point $P = (11, 2, 1)$ and is perpendicular to the line of intersection of planes $x + y - z - 2 = 0$ and $2x - y + 3z - 1 = 0$.

Problem 12. (3 points) Determine whether the line of parametric equations

$$x = 1 + 2t, \quad y = -2t, \quad z = 2 + t, \quad t \in \mathbb{R}$$

intersects the plane with equation $3x + 4y + 6z - 7 = 0$. If it does intersect, find the point of intersection.

Problem 13. (3 points) Evaluate $\lim_{t\to 0}\langle e^t, \frac{\sin(t)}{t}, e^{-t}\rangle$.

Problem 14. (3 points) Compute the derivative of the vector-valued function $\mathbf{r}(t) = t^3\mathbf{i} + 3t^2\mathbf{j} + \frac{t^3}{6}\mathbf{k}$.

Problem 15. (3 points) Evaluate the integral $\int (t\mathbf{i} + 2t\mathbf{j} - \cos(t)\mathbf{k})\, dt$.

Problem 16. (5 points) Find the arc-length parameterization for the curve

$$\mathbf{r}(t) = \langle 3t - 2, -t, 2t + 1 \rangle$$

for $t \geq 3$.

Problem 17. (5 points) Find the curvature of $\mathbf{r}(t) = \langle \sqrt{2}t, e^t, e^-t \rangle$ at the point $(0, 1, 1)$.

The End