University of South Carolina
MATH 241-H01

Practice Midterm Examination A

February 2, 2023

Closed book examination — Time: 75 minutes

Instructions:

Notes, books, computer, phones, calculators or other aids are **not** allowed. Please write on only one side of each page. If you need more space than is provided, then ask for extra paper from the proctor. Simplify your final answers. Full credit will not be awarded for insufficient accompanying work.

There are 55 points available, but the exam is **out of** 50.
(In other words, there are 5 bonus points available)

Problem 1. (3 points) Find a vector $\mathbf{v}$ with magnitude 7 and in the same direction as $\langle 4, 6 \rangle$

Problem 2. (3 points) Given vectors $\mathbf{a} = \langle 2, 5, -1 \rangle$ and $\mathbf{b} = \langle 2, -1, 1 \rangle$, express the vectors $\mathbf{a} + \mathbf{b}$, $4\mathbf{a}$ and $-5\mathbf{a} + 3\mathbf{b}$ in component form.

Problem 3. (3 points) Find the component form of the two-dimensional vector $\mathbf{u}$ where $\|\mathbf{u}\| = 4$ and the angle the vector makes with the positive direction of the x-axis is $60°$ in a clockwise direction.

Problem 4. (3 points) Given vectors $\mathbf{a} = \langle 4, 1, 2\rangle$, $\mathbf{b} = \langle 1, 5, 0\rangle$, and $\mathbf{c} = \langle 1, 5, 0\rangle$, compute $(\mathbf{a} \cdot \mathbf{b})\mathbf{c}$.

Problem 5. (3 points) Determine the real number α such that the vectors $\mathbf{a} = 2\mathbf{i} + \alpha\mathbf{j}$ and $\mathbf{b} = 5\mathbf{i} - 3\mathbf{j}$ are orthogonal.

Problem 6. (3 points) Simplify $(\mathbf{j} \times \mathbf{i} - 3\mathbf{i} \times \mathbf{j} + \mathbf{i} \times \mathbf{k} + 3\mathbf{k} \times \mathbf{k}) \cdot \mathbf{j}$.

Problem 7. (3 points) Find the vector projection of $\mathbf{v} = \langle 3, 2, 1 \rangle$ onto $\mathbf{u} = \langle 2, -1, 3 \rangle$.

Problem 8. (3 points) Find the area of the parallelogram with adjacent sides $\mathbf{u} = \langle 1, 1, 0 \rangle$ and $\mathbf{v} = \langle 2, -1, 1 \rangle$.

Problem 9. (3 points) Find a normal vector $\mathbf{n}$ to the plane with equation $2x - y + 13z - 10 = 0$.

Problem 10. (3 points) Find the distance between the point $(1, 1, 3)$ and the line given by the equations $x - 3 = 2y + 2 = 4z - 12$.

Problem 11. (3 points) Find the general equation of the plane passing through $P = (1, 0, 1)$, $Q = (2, 4, 0)$, and $R = (1, 1, -1)$.

Problem 12. (3 points) Find parametric equations for the line formed by the intersection of planes $x + y - z = 3$ and $3x - y + 3z = 5$.

Problem 13. (3 points) Find the tangent vector to $\mathbf{r}(t) = \langle 3t^3, 2t^2, \frac{1}{t} \rangle$ at $t = 1$.

Problem 14. (3 points) Given $\mathbf{r}(t) = \langle t, t^2, -t^4 \rangle$, find $\frac{d}{dt}\left[r(t^2)\right]$.

Problem 15. (3 points) Evaluate the integral $\int \left(e^t\mathbf{i} + t^2\mathbf{j} - \sin(t)\mathbf{k}\right) dt$.

Problem 16. (5 points) Compute the arc length of the parametrized curve

$$\mathbf{r}(t) = \langle 2t^2 + 1, 2t^2 - 1, t^3 \rangle$$

for $0 \leq t \leq 2$.

Problem 17. (5 points) Find the principal unit tangent vector and the principal unit normal vector for the curve

$$\mathbf{r}(t) = \langle t^3 - 4t, 5t^2 - 2 \rangle$$

for $t \geq 3$.

The End