University of South Carolina
MATH 241-H01

Midterm Examination 1

February 2, 2023

Closed book examination Time: 75 minutes

Instructions:

Notes, books, computer, phones, calculators or other aids are **not** allowed. Please write on only one side of each page. If you need more space than is provided, then ask for extra paper from the proctor. Simplify your final answers. Full credit will not be awarded for insufficient accompanying work.

There are 55 points available, but the exam is **out of** 50.
(In other words, there are 5 bonus points available)

Problem 1. (3 points) Given vectors $\mathbf{a} = \langle 2, 5, -1 \rangle$ and $\mathbf{b} = \langle 2, -1, 1 \rangle$, express the vector $5\mathbf{a} - \mathbf{b}$ in component form.

Problem 2. (3 points) A vector $\mathbf{v}$ has initial point $(1, 2, 4)$ and terminal point $(0, 2, 2)$. Find the unit vector in the direction of $\mathbf{v}$. Express the answer in component form.

Problem 3. (3 points) Find the component form of the two-dimensional vector $\mathbf{u}$ where $\|\mathbf{u}\| = 3$ and the angle the vector makes with the positive direction of the x-axis is $30°$ in a counter-clockwise direction.

Problem 4. (3 points) Given vectors $\mathbf{a} = \mathbf{i} + \mathbf{k}$, $\mathbf{b} = 2\mathbf{i} - \mathbf{j}$, and $\mathbf{c} = \mathbf{j} + \mathbf{k}$, compute $(\mathbf{a} \cdot \mathbf{b})\mathbf{c}$.

Problem 5. (3 points) Given vectors $\mathbf{a} = \langle 1, 2, 3 \rangle$ and $\mathbf{b} = \langle 0, 2, 1 \rangle$, compute $\mathbf{a} \times \mathbf{b}$.

Problem 6. (3 points) Find a vector $\mathbf{c}$ that is orthogonal to both $\mathbf{a} = \langle 1, 1, 1 \rangle$ and $\mathbf{b} = \langle 1, 2, 0 \rangle$.

Problem 7. (3 points) Determine the vector projection $\text{proj}_{\mathbf{u}}\,\mathbf{v}$ of the vector $\mathbf{v} = \langle 2, 1, 1\rangle$ onto the vector $\mathbf{u} = \langle 1, 1, 1\rangle$.

Problem 8. (3 points) Find a normal vector $\mathbf{n}$ to the plane with equation $x - 2y + 3z - 60 = 0$.

Problem 9. (3 points) Let L be the line passing through the point $P = (0, 1, 1)$ with direction $\mathbf{v} = \langle 3, 2, 1\rangle$. Find symmetric equations of the line L.

Problem 10. (3 points) Find the general equation of the plane passing through $P = (0, 0, 1)$, $Q = (2, 3, 0)$, and $R = (1, 0, -1)$.

Problem 11. (3 points) Find the distance from the point $P = (4, 1, 1)$ to the plane with equation $2x - y + z = 0$.

Problem 12. (3 points) Determine whether the line with parametric equations

$$x = 1 + t, \quad y = 1 - t, \quad z = 2 + t, \quad t \in \mathbb{R}$$

intersects the plane with equation $x + 2y + z - 10 = 0$. If it does intersect, find the point of intersection.

Problem 13. (3 points) Evaluate $\lim_{t\to 1}\left\langle \frac{t^2-1}{t-1}, \ln(t), e^{-t}\right\rangle$.

Problem 14. (3 points) Find the tangent vector to $\mathbf{r}(t) = \langle t^2, \ln(t), e^t\rangle$ at $t = 1$.

Problem 15. (3 points) Evaluate the integral $\int \left(t^2\mathbf{i} + \cos(t)\mathbf{j} - e^t\mathbf{k}\right) dt$.

Problem 16. (5 points) Determine the length of the parametric curve given by $x = 3t^2$, $y = 2t^3$ where $0 \leq t \leq 1$.

Problem 17. (5 points) Find the principal unit tangent vector and the principal unit normal vector for the curve

$$\mathbf{r}(t) = \langle \sin(t), \cos(t), 1 - 3t \rangle$$

at $t = 0$.

The End