University of South Carolina
MATH 241-H01

Final Examination Practice B

May 2, 2023

Closed book examination Time: 150 minutes

Instructions:

Notes, books, computer, phones, calculators or other aids are **not** allowed. Please write on only one side of each page. If you need more space than is provided, then ask for extra paper from the proctor. Simplify your final answers. Full credit will not be awarded for insufficient accompanying work.

There are 99 points available, but the exam is **out of** 90.
(In other words, there are 90 bonus points available)

Problem 1. (3 points) A vector $\mathbf{v}$ has initial point $(2, 3, 1)$ and terminal point $(1, 0, 2)$. Find the unit vector in the direction of $\mathbf{v}$.

Problem 2. (3 points) Given vectors $\mathbf{a} = 2\mathbf{i} + \mathbf{j}$, $\mathbf{b} = 3\mathbf{i} - 2\mathbf{j}$, and $\mathbf{c} = \mathbf{j} + \mathbf{k}$, compute $(\mathbf{a} \cdot \mathbf{b})\mathbf{c}$.

Problem 3. (3 points) Given vectors $\mathbf{a} = \langle 1, 2, 3\rangle$ and $\mathbf{b} = \langle 0, 2, 1\rangle$, compute $\mathbf{a} \times \mathbf{b}$.

Problem 4. (3 points) Find the general equation of the plane passing through $P = (0, 0, 1)$, $Q = (2, 1, 0)$, and $R = (1, 1, 1)$.

Problem 5. (3 points) Determine the vector projection $\text{proj}_{\mathbf{u}}\,\mathbf{v}$ of the vector $\mathbf{v} = \langle 5, 2 \rangle$ onto the vector $\mathbf{u} = \langle 3, 4 \rangle$.

Problem 6. (3 points) Find the area of the parallelogram with adjacent edges $\mathbf{u} = \langle -3, 1 \rangle$, and $\mathbf{v} = \langle 2, 1 \rangle$.

Problem 7. (3 points) Determine the real number α such that the vectors $\mathbf{a} = 3\mathbf{i} + \alpha\mathbf{j}$ and $\mathbf{b} = \mathbf{i} - \mathbf{j}$ are orthogonal.

Problem 8. (3 points) Determine whether the line with parametric equations

$$x = 1, \quad y = t, \quad z = 2 + 2t, \quad t \in \mathbb{R}$$

intersects the plane with equation $x + y + z - 10 = 0$. If it does intersect, find the point of intersection.

Problem 9. (3 points) Compute the derivative of the vector-valued function $\mathbf{r}(t) = t\mathbf{i} + t^3\mathbf{j} + \cos(t)\mathbf{k}$.

Problem 10. (3 points) Evaluate the integral $\displaystyle\int \left(\mathbf{i} + t\mathbf{j} - \frac{1}{1+t^2}\mathbf{k}\right) dt$.

Problem 11. (5 points) Find the principal unit tangent vector and the principal unit normal vector for the curve

$$\mathbf{r}(t) = \langle \cos(t), \sin(t), 2t \rangle$$

at $t = 0$.

Problem 12. (5 points) Find and sketch the level curves $f(x, y) = c$ on the same set of axes for the function

$$f(x, y) = 2(x - 1)^2 + 2y^2$$

at the values $c = 0, 2, 8$.

Problem 13. (3 points) Find the limit $\displaystyle\lim_{(x,y)\to(1,1)} \frac{x^2 - y^2}{x - y}$.

Problem 14. (4 points) Show the limit $\lim_{(x,y)\to(0,0)} \dfrac{xy - y^3}{x^2 + y^2}$ does not exist by considering the paths $y = -x$ and $y = x$.

Problem 15. (4 points) Given $f(x, y) = \sqrt{x^2 + y^2}$, approximate $f(4.1, 2.9)$ using a linear approximation at the point $(4, 3)$.

Problem 16. (8 points) Find the absolute extrema of the function

$$f(x, y) = 2x^2 - 3xy + 2y^2$$

on the region bounded by the unit circle centered at the origin.

Problem 17. (8 points) Find the minimum and maximum distances between the ellipse $x^2+xy+2y^2=1$ and the origin.

Problem 18. (4 points) Find $\displaystyle\int_1^3 \int_{\sqrt{y}}^{1-y} 1 - 2x \; dx \; dy$.

Problem 19. (4 points) Express the following integral using spherical coordinates

$$\iiint_E f(x,y,z)dV$$

where E is the region such that $1 \leq x^2 + y^2 + z^2 \leq 3$, $z \geq 0$ and $y \geq 0$.

Problem 20. (8 points) Find the volume of the solid bounded by the planes $x = 0$, $y = 0$, $z = 0$ and $x + y + z = 2$.

Problem 21. (4 points) Sketch the vector field $\mathbf{F} = \langle y, -x \rangle$.

Problem 22. (4 points) Determine whether the vector field $\mathbf{F} = \langle 2xy, x^2 + y^2 \rangle$ is conservative and, if so, find a potential function.

Problem 23. (8 points) Compute the area of the region bounded by the hypocycloid $\mathbf{r}(t) = \cos^3(t)\mathbf{i} + \sin^3(t)\mathbf{j}$ parameterized by $t \in [0, 2\pi]$.

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