University of South Carolina
MATH 241-H01

Final Examination Practice A

May 2, 2023

Closed book examination | Time: 150 minutes

Instructions:

Notes, books, computer, phones, calculators or other aids are **not** allowed. Please write on only one side of each page. If you need more space than is provided, then ask for extra paper from the proctor. Simplify your final answers. Full credit will not be awarded for insufficient accompanying work.

There are 99 points available, but the exam is **out of** 90.
(In other words, there are 90 bonus points available)

Problem 1. (3 points) Given vectors $\mathbf{a} = \langle 2, 2, 1 \rangle$ and $\mathbf{b} = \langle 3, -1, 4 \rangle$, express the vector $2\mathbf{a} + \mathbf{b}$ in component form.

Problem 2. (3 points) Find the component form of the two-dimensional vector $\mathbf{u}$ where $\|\mathbf{u}\| = 2$ and the angle the vector makes with the positive direction of the x-axis is $60°$ in a counter-clockwise direction.

Problem 3. (3 points) Find a vector $\mathbf{c}$ that is orthogonal to both $\mathbf{a} = \langle 2, 3, -1 \rangle$ and $\mathbf{b} = \langle 3, 0, 2 \rangle$.

Problem 4. (3 points) Find a normal vector $\mathbf{n}$ to the plane with equation $x + 2y + 3z = 4$.

Problem 5. (3 points) Let L be the line passing through the point $P = (1, 0, 1)$ with direction $\mathbf{v} = \langle 2, 2, 1 \rangle$. Find symmetric equations of the line L.

Problem 6. (3 points) Determine whether the line of parametric equations

$$x = 2 + t, \quad y = -t, \quad z = 1 + t, \quad t \in \mathbb{R}$$

intersects the plane with equation $x - 2y + 4z = 7$. If it does intersect, find the point of intersection.

Problem 7. (3 points) Find the distance from the point $P = (2, 1, 2)$ to the plane with equation $x - y + z = 0$.

Problem 8. (3 points) Evaluate $\lim_{u \to 2} \left\langle \frac{u^2 - 4}{u - 2}, u^3, \frac{1}{u} \right\rangle$.

Problem 9. (3 points) Find the tangent vector to $\mathbf{r}(t) = \langle 2t, 3t^2, 2^t \rangle$ at $t = 2$.

Problem 10. (3 points) Given $\mathbf{r}(t) = \langle 1, 2t, -t^2 \rangle$, find $\dfrac{d}{dt}\left[\mathbf{r}(t^2)\right]$.

Problem 11. (5 points) Find the arc-length parameterization for the curve

$$\mathbf{r}(t) = \langle 3t - 2, 1 - 2t \rangle$$

for $t \geq 5$.

Problem 12. (5 points) Find and sketch the level curves $f(x, y) = c$ on the same set of axes for the function

$$f(x, y) = y(x - 1)$$

at the values $c = -2, -1, 0, 1, 2$.

Problem 13. (3 points) Find the limit $\lim_{(x,y)\to(2,2)} \frac{e^{y+x}}{x - y^2}$.

Problem 14. (4 points) Find the directional derivative of the function

$$f(x, y) = \cos(x) + \cos(y)$$

at the point $(0, 0)$ in the direction of $\mathbf{u} = \left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle$.

Problem 15. (4 points) Find the equation for the tangent plane to the surface

$$z = \cos(xy) + y$$

for $(x, y) = (0, 0)$.

Problem 16. (8 points) For the function

$$f(x, y) = 2x^2 - 3y^3 + 2xy,$$

find all the critical points and use the second derivative test to determine, if possible, whether each is a maximum, minimum, or saddle point.

Problem 17. (8 points) Find the absolute extrema of the function

$$f(x, y) = 3xy + y^2$$

on the closed and bounded rectangle given by $0 \leq x \leq 1$ and $0 \leq y \leq 1$.

Problem 18. (4 points) Find $\int_{-1}^{1}\int_{0}^{1}\int_{0}^{2} x^2y - 2z\; dx\; dy\; dz$.

Problem 19. (4 points) Change the order of integration in

$$\int_{0}^{9}\int_{\sqrt{x}}^{3} f(x,y)\; dy\; dx.$$

(This may involve breaking the integral up into multiple integrals.)

Problem 20. (8 points) Find $\iiint_E x^2 + y^2 \, dV$ where E is the solid region satisfying the inequalities $0 \leq x^2 + y^2 \leq 4$, $y \geq 0$, and $0 \leq z \leq 3 - x$.

Problem 21. (4 points) Evaluate the line integral $\int_C yz\,dx + xy\,dy + z^2\,dz$ along the curve C parameterized by $\mathbf{r}(t) = (t, t^2, t^3)$ for $0 \leq t \leq 1$.

Problem 22. (4 points) Compute the flux of $\mathbf{F} = x^2\mathbf{i} + y\mathbf{j}$ across a line segment from $(0, 0)$ to $(1, 2)$.

Problem 23. (8 points) Evaluate $\oint_C y^3\ dx - x^3y^2\ dy$ where C is a circle of radius 2 centered at the origin oriented counterclockwise.

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