University of South Carolina

Midterm Examination 3 November 21, 2017

Math 142–005/006

Closed book examination Time: 75 minutes

Name ________________________________

Instructions:

No notes, books, or calculators are allowed. If you need more space than is provided use the back of the previous page and clearly indicate you have done so. Simplify your final answers. **Full credit may not be awarded for insufficient accompanying work.**

1		12
2		8
3		8
4		6
5		9
6		7
Total		50

1. (12 points) For each of the following functions:

- write down the Maclaurin series using Σ notation, and
- write down the radius of convergence.

(You do not need to justify your answers.)

(a) e^x

(b) $\cos(x)$

(c) $(1+x)^{\frac{1}{2}}$

(d) $\tan^{-1}(x)$

2. (8 points) Determine the Taylor polynomial of order 3 generated by the function $f(x) = \dfrac{1}{x^2}$ at $x = 1$.

3. (8 points) Determine the interval of convergence for the power series

$$\sum_{n=1}^{\infty} \frac{(4x-5)^n}{n} \, .$$

4. (6 points)

(a) Using the Maclaurin polynomial of order 4 for $f(x) = e^x$, estimate the value of $\frac{1}{e}$.

(b) What is the maximum value of $|f^{(5)}(x)|$ on the interval $[-1, 0]$?

(c) Find an upper bound on the absolute value of the error for the estimate from (a) using the Remainder Estimation Theorem.

5. (9 points) Find the following:

(a) The Taylor polynomial of order 3 generated by $f(x) = \ln(x+1)/x$ at $x = 0$.

(b) The Taylor polynomial of order 9 generated by $f(x) = e^{-x^3}$ at $x = 0$.

(c) $\displaystyle\lim_{x \to 0} \frac{\cos(x) - 1 + \frac{x^2}{2}}{x^4}$

<u>This question requires material not tested in 2018 Midterm 3.</u>

6. (7 points) Let C be the parametric curve determined by

$$x = t^2$$
$$y = t^3 + t$$

where t is a parameter in the interval $[0, 2]$.

(a) Determine the x and y coordinates of the point when $t = 1$.

(b) Determine $\left.\dfrac{dy}{dx}\right|_{t=1}$.

(c) Find an equation for the line tangent to the curve C at the point where $t = 1$.

(d) Determine $\left.\dfrac{d^2y}{dx^2}\right|_{t=1}$.

The End