

**University of South Carolina**  
Midterm Examination 3    October 20, 2016  
**Math 142 Section H03**

Closed book examination

Time: 75 minutes

**Name** \_\_\_\_\_

**Instructions:**

No notes, books, or calculators are allowed. If you need more space than is provided use the back of the previous page and clearly indicate you have done so. Simplify your final answers. **Full credit may not be awarded for insufficient accompanying work.**

1		8
2		8
3		8
4		10
5		8
6		8
Total		50

1. (8 points) For each of the following functions:

- write down the Maclaurin series using  $\Sigma$  notation, and
- write down their interval of convergence.

(You do not need to justify your answers.)

(a)  $e^x$

(b)  $\sin(x)$

(c)  $(1+x)^{\frac{1}{3}}$

(d)  $\ln(1+x)$

2. (8 points) Determine the Taylor polynomial of order 4 generated by the function  $\cos^2(x)$  at  $x = \pi$ .

3. (8 points) Determine the interval of convergence for the power series

$$\sum_{n=0}^{\infty} \frac{(x-5)^n}{2n+1}.$$

4. (10 points)

(a) Using the Taylor polynomial of order 2 generated by the function  $f(x) = \sqrt{x}$  at  $x = 4$ , estimate the value of  $\sqrt{5}$ .

(b) What is the maximum value of  $|f^{(3)}(x)|$  on the interval  $[4, 5]$ ?

(c) Find an upper bound on the absolute value of the error for the estimate from (a) using the Remainder Estimation Theorem.

5. (8 points) Find the following:

(a)  $\lim_{x \rightarrow 0} \frac{\tan^{-1}(x) - x}{\sin(x) - x}$

(b)  $\sum_{n=0}^{\infty} \left(\frac{1}{3}\right)^n$

(c)  $\sum_{n=0}^{\infty} \frac{4^n}{n!}$

6. (8 points) Use the Taylor polynomial of order 3 generated by  $\sin(x)$  at  $x = 0$  to estimate

$$\int_0^3 \frac{\sin(2x)}{x} dx .$$