University of South Carolina

Midterm Examination 2 November 4, 2021

Math 142–003/004

Closed book examination Time: 75 minutes

Name _____

Instructions:

Notes, books, computer, phones, calculators or other aids are **not** allowed. Please write on only one side of each page. If you need more space than is provided, then ask for extra paper from the proctor. Simplify your final answers. Full credit will not be awarded for insufficient accompanying work.

There are $9 + 9 + 12 + 8 + 10 + 12 = 60$ points available, but the exam is **out of** 55.
(In other words, there are 5 bonus points available)

1. (9 points) For each series, what can you conclude from the given convergence test?

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{3^n}{n^2}$ using the Root Test.

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{3^n}{n!}$ using the Ratio Test.

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{2n}{1+n^2}$ using the Integral Test.

2. (9 points) For each series, what can you conclude from the given convergence test?

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{\ln(n)}{n^2+n}$ using the Limit Comparison Test with $\displaystyle\sum \frac{1}{n}$.

(b) $\displaystyle\sum_{n=4}^{\infty} \frac{2n+1}{n^3+1}$ using the Limit Comparison Test with $\displaystyle\sum \frac{1}{n^2}$.

(c) $\displaystyle\sum_{n=4}^{\infty} \frac{3n^2+1}{n^2+2}$ using the Limit Comparison Test with $\displaystyle\sum \frac{1}{n^2}$.

3. (12 points) For each of the following series, determine if it converges or diverges.

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{2^{2n}}{(n+1)^n}$

(b) $\displaystyle\sum_{n=3}^{\infty} \frac{3n^2+1}{n^4-4}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{5^n}{(2n)!}$

4. (8 points) For each of the following series, determine if it

   - converges absolutely,
   - converges conditionally, or
   - diverges.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n+1}$

(b) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^{n+1} n}{n+1}$

5. (10 points) Determine the interval of convergence for the power series

$$\sum_{n=2}^{\infty} \frac{(x-3)^n}{n5^n}.$$

6. (12 points) The power series

$$f(x) = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + x^4 + \cdots$$
$$g(x) = \sum_{n=0}^{\infty} nx^n = x + 2x^2 + 3x^3 + 4x^4 + 5x^5 + \cdots$$

converge for $-1 < x < 1$ (you don't need to show this). Find the first 4 non-zero terms of the following power series:

(a) $f(x) + g(x)$

(b) $g(2x^2)$

(c) $f'(x)$

(d) $\displaystyle\int_0^x f(y)\, dy$

The End