

University of South Carolina

Midterm Examination 2    November 5, 2020

Math 142–011/012/031/032

Closed book examination

Time: 75 minutes

Name \_\_\_\_\_

**Instructions:**

No notes, books, computer, phones, calculators or other aids are allowed. You must be alone while you are taking the test and you should not be in contact electronically or physically with any other person from the time you start the exam until you submit your final solutions. Do not use more time than the time allotted. Full credit will not be awarded for insufficient accompanying work.

Submit your completed exam on Blackboard by uploading a scanned PDF file or as multiple JPEG image files. You do not have to print the exam, but please have a separate page/image for each page of the exam. Clearly indicate which questions you are answering on every image/page.

1		12
2		12
3		12
4		8
5		8
6		8
Total		60

1. (12 points) For each series, what can you conclude from the given convergence test?

(a)  $\sum_{n=4}^{\infty} \frac{1}{\ln(n)}$  using the Limit Comparison Test with  $\sum \frac{1}{n}$ .

(b)  $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$  using the Integral Test.

(c)  $\sum_{n=1}^{\infty} \left(\frac{4n+3}{3n-5}\right)^n$  using the Root Test.

2. (12 points) For each series, what can you conclude from the given convergence test?

(a)  $\sum_{n=1}^{\infty} \frac{2^{n+1}}{n3^{n-1}}$  using the Ratio Test.

(b)  $\sum_{n=1}^{\infty} (-1)^n \frac{10^n}{n+1}$  using the Alternating Series Test.

(c)  $\sum_{n=10}^{\infty} \frac{n+1}{2n^3-2}$  using the Direct Comparison Test with  $\sum \frac{1}{n^2}$ .

3. (12 points) For each of the following series, determine if it converges or diverges.

(a) 
$$\sum_{n=3}^{\infty} \frac{n}{\ln(n)}$$

(b) 
$$\sum_{n=1}^{\infty} \frac{3^{n+1}}{(2n+1)!}$$

(c) 
$$\sum_{n=1}^{\infty} \frac{2^n}{3+4^n}$$

4. (8 points) Determine the interval of convergence for the power series

$$\sum_{n=1}^{\infty} \frac{(x-1)^n}{n^3 3^n}.$$

5. (8 points) Determine the Taylor polynomial of order 4 generated by  $f(x) = \cos^2(x)$  at  $x = 0$ .

6. (8 points)

(a) Using the Taylor polynomial of order 2 generated by the function  $f(x) = \ln(x)$  at  $x = 1$ , estimate the value of  $\ln(\frac{1}{3})$ .

(b) What is the maximum value of  $|f^{(3)}(x)|$  on the interval  $[\frac{1}{3}, 1]$ ?

(c) Find an upper bound on the absolute value of the error for the estimate from (a) using the Remainder Estimation Theorem.

**The End**