

**University of South Carolina**  
Midterm Examination 2    October 24, 2016  
**Math 142–005/006**

Closed book examination

Time: 75 minutes

**Name** \_\_\_\_\_

**Instructions:**

No notes, books, or calculators are allowed. If you need more space than is provided use the back of the previous page and clearly indicate you have done so. Simplify your final answers. **Full credit may not be awarded for insufficient accompanying work.**

1		9
2		9
3		9
4		9
5		12
6		8
Total		56

1. (9 points) Find the limit of each of the following sequences or explain why the limit does not exist.

(a)  $\lim_{n \rightarrow \infty} \frac{n^3 - 2n + 1}{n^3 + n - 1}$

(b)  $\lim_{n \rightarrow \infty} \frac{n^3}{2^n}$

(c)  $\lim_{n \rightarrow \infty} (7n)^{2/n}$

2. (9 points) Find the value of each of the following series or explain why the series diverges.

(a)  $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$

(b)  $\sum_{n=1}^{\infty} \left(\frac{1}{3}\right)^n$

(c)  $\sum_{n=0}^{\infty} \frac{2^n - 5}{3^n}$

3. (9 points) For each series, what can you conclude from the given convergence test?

(a)  $\sum_{n=1}^{\infty} \frac{1}{n^4}$  using the Integral Test.

(b)  $\sum_{n=1}^{\infty} \frac{4^n}{n!}$  using the Ratio Test.

(c)  $\sum_{n=1}^{\infty} \frac{2^n}{n^4}$  using the Root Test.

4. (9 points) For each series, what can you conclude from the given convergence test?

(a)  $\sum_{n=1}^{\infty} \frac{3}{n^2 + 1}$  using the Limit Comparison Test with  $\sum \frac{1}{n^2}$ .

(b)  $\sum_{n=4}^{\infty} \frac{1}{n + 1}$  using the Limit Comparison Test with  $\sum \frac{1}{n^2}$ .

(c)  $\sum_{n=2}^{\infty} \frac{1}{n - 1}$  using the Direct Comparison Test with  $\sum \frac{1}{n}$ .

5. (12 points) For each of the following series, determine if it converges or diverges.

(a) 
$$\sum_{n=0}^{\infty} \frac{2}{n^n}$$

(b) 
$$\sum_{n=3}^{\infty} \frac{n^3 - 2n + 1}{n^4 - 4n + 2}$$

(c) 
$$\sum_{n=1}^{\infty} \frac{n^3}{n!}$$

6. (8 points) For each of the following series, determine if it

- converges absolutely,
- converges conditionally, or
- diverges.

(a)  $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^3}$

(b)  $\sum_{n=2}^{\infty} \frac{(-1)^{n+1}}{\ln(n)}$