University of South Carolina

Midterm Examination 2 October 24, 2016

Math 142–003/004

Closed book examination	Time: 75 minutes
Name	

Instructions:

No notes, books, or calculators are allowed. If you need more space than is provided use the back of the previous page and clearly indicate you have done so. Simplify your final answers. Full credit may not be awarded for insufficient accompanying work.

1	9
2	9
3	9
4	9
5	12
6	8
Total	56

1. (9 points) Find the limit of each of the following sequences or explain why the limit does not exist.

(a)
$$\lim_{n \to \infty} \frac{4n^2 + 2n - 1}{n^3 + 1}$$

(b)
$$\lim_{n \to \infty} \frac{n^2}{2^n}$$

(c)
$$\lim_{n\to\infty} (3n)^{4/n}$$

2. (9 points) Find the value of each of the following series or explain why the series diverges.

(a)
$$\sum_{n=1}^{\infty} n$$

(b)
$$\sum_{n=1}^{\infty} \left(\frac{1}{3}\right)^n$$

(c)
$$\sum_{n=0}^{\infty} \frac{3^n - 2}{4^n}$$

- 3. (9 points) For each series, what can you conclude from the given convergence test?
 - (a) $\sum_{n=1}^{\infty} e^{-n}$ using the Integral Test.

(b) $\sum_{n=1}^{\infty} \frac{2^n}{n!}$ using the Ratio Test.

(c) $\sum_{n=1}^{\infty} \frac{3^n}{n^3}$ using the Root Test.

- 4. (9 points) For each series, what can you conclude from the given convergence test?
 - (a) $\sum_{n=1}^{\infty} \frac{3}{n+1}$ using the Limit Comparison Test with $\sum_{n=1}^{\infty} \frac{1}{n}$.

(b) $\sum_{n=4}^{\infty} \frac{1}{n^2+1}$ using the Limit Comparison Test with $\sum_{n=1}^{\infty} \frac{1}{n}$.

(c) $\sum_{n=2}^{\infty} \frac{1}{n-1}$ using the Direct Comparison Test with $\sum_{n=2}^{\infty} \frac{1}{n}$.

5. (12 points) For each of the following series, determine if it converges or diverges.

(a)
$$\sum_{n=0}^{\infty} \frac{2}{(n+1)^n}$$

(b)
$$\sum_{n=3}^{\infty} \frac{n^2 + 2n - 1}{n^4 - 2n + 3}$$

(c)
$$\sum_{n=1}^{\infty} \frac{n^2}{(2n)!}$$

- 6. (8 points) For each of the following series, determine if it
 - converges absolutely,
 - converges conditionally, or
 - diverges.

(a)
$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$$

(b)
$$\sum_{n=2}^{\infty} \frac{(-1)^{n+1}n}{\ln(n)}$$