University of South Carolina

Midterm Examination 1 October 1, 2020

Math 142–011/012/031/032

Closed book examination Time: 75 minutes

Name Solutions

Instructions:

No notes, books, computer, phones, calculators or other aids are allowed. You must be alone while you are taking the test and you should not be in contact electronically or physically with any other person from the time you start the exam until you submit your final solutions. Do not use more time than the time allotted. Full credit will not be awarded for insufficient accompanying work.

Submit your completed exam on Blackboard by uploading a scanned PDF file or as multiple JPEG image files. You do not have to print the exam, but please have a separate page/image for each page of the exam. Clearly indicate which questions you are answering on every image/page.

1	9	9
2	9	9
3	10	10
4	10	10
5	12	12
6	10	10
Total	60	60

1. (9 points) Find the following integrals.

(a) $\displaystyle\int \frac{4x^3}{1+x^4}\,dx$

$$\boxed{\begin{aligned} u &= 1+x^4 \\ du &= 4x^3\,dx \end{aligned}} \Rightarrow \int \frac{du}{u} = \ln|u| + C$$

$$= \ln(1+x^4) + C$$

(b) $\displaystyle\int 2t\cos(3t)\,dt$

$$\boxed{\begin{aligned} u &= 2t & v &= \tfrac{1}{3}\sin(3t) \\ du &= 2\,dt & dv &= \cos(3t)\,dt \end{aligned}} \Rightarrow$$

$$= (2t)\left(\frac{1}{3}\sin(3t)\right) - \int \frac{1}{3}\sin(3t)\,2\,dt$$

$$= \frac{2}{3}t\sin(3t) + \frac{2}{9}\cos(3t) + C$$

(c) $\displaystyle\int \tan^2(\theta)\sec^2(\theta)\,d\theta$

$$\boxed{\begin{aligned} u &= \tan\theta \\ du &= \sec^2\theta\,du \end{aligned}} = \int u^2\,du = \frac{1}{3}u^3 + C$$

$$= \frac{1}{3}\tan^3\theta + C$$

2. (9 points) Find the following integrals.

(a) $\int \cos^3(5x)\,dx$ $= \int (1-\sin^2(5x))\cos(5x)\,dx$

$$\boxed{\begin{aligned} u &= \sin(5x) \\ du &= 5\cos(5x)\,dx \end{aligned}}$$

$$= \int \frac{1-u^2}{5}\,du = \frac{u}{5} + \frac{u^3}{15} + C$$

$$= \frac{\sin(5x)}{5} - \frac{\sin^3(5x)}{15} + C$$

(b) $\int \frac{3}{x^2+6x+10}\,dx$

$$= 3\int \frac{1}{(x+3)^2+1}\,dx \quad \boxed{\begin{aligned} u &= x+3 \\ du &= dx \end{aligned}} \quad = 3\int \frac{1}{u^2+1}\,du$$

$$= 3\arctan(u) + C = 3\arctan(x+3) + C$$

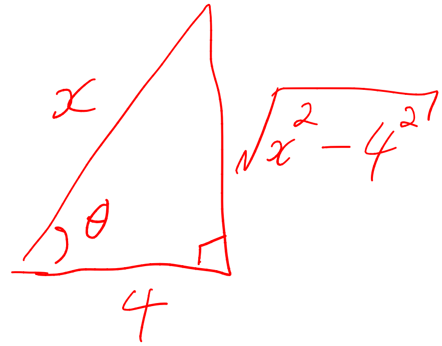
(c) $\int x^2 e^{2x}\,dx$

$$\boxed{\begin{aligned} u &= x^2 & v &= \tfrac{1}{2}e^{2x} \\ du &= 2x & dv &= e^{2x}\,dx \end{aligned}} \quad = \frac{x^2}{2}e^{2x} - \int x e^{2x}\,dx$$

$$\boxed{\begin{aligned} u &= x & v &= \tfrac{1}{2}e^{2x} \\ du &= dx & dv &= e^{2x}\,dx \end{aligned}} \quad = \frac{x^2}{2}e^{2x} - \frac{x}{2}e^{2x} + \int \frac{1}{2}e^{2x}\,dx$$

$$= \left(\frac{x^2}{2} - \frac{x}{2} + \frac{1}{4}\right)e^{2x} + C$$

3. (10 points) Find $\int \frac{1}{\sqrt{x^2-16}}\,dx$ for $x > 4$.



$$\boxed{\begin{aligned} x &= 4\sec\theta \\ dx &= 4\sec\theta\tan\theta\,d\theta \\ \sqrt{x^2-16} &= 4\tan\theta \end{aligned}}$$

$$= \int \frac{4\sec\theta\tan\theta\,d\theta}{4\tan\theta} = \int \sec\theta\,d\theta$$

$$= \ln\left|\sec\theta + \tan\theta\right| + C$$

$$= \ln\left|\frac{x}{4} + \frac{\sqrt{x^2-16}}{4}\right| + C$$

[Optionally: $= \ln\left(x + \sqrt{x^2-16}\right) + C$]

4. (10 points) Find the following integrals.

(a) $\displaystyle\int \frac{3x^2+2x+1}{(x+1)(x^2+1)}\,dx = \int \frac{A}{x+1} + \frac{Bx+C}{x^2+1}\,dx$

Must solve $3x^2+2x+1 = (A+B)x^2 + (B+C)x + (A+C)$

$$\boxed{\begin{aligned} A+B&=3\\ B+C&=2\\ A+C&=1\end{aligned}} \Rightarrow \boxed{\begin{aligned} A&=1\\ B&=2\\ C&=0\end{aligned}}$$

$$= \int \frac{1}{x+1} + \frac{2x}{x^2+1}\,dx$$

$$= \ln|x+1| + \ln|x^2+1| + C$$

(b) $\displaystyle\int_0^\infty te^{-t}\,dt$

$$\int te^{-t}\,dt = t(-e^{-t}) - \int -e^{-t}\,dt$$

$$\boxed{\begin{aligned} u&=t & v&=-e^{-t}\\ du&=dt & dv&=e^{-t}dt\end{aligned}}$$

$$= -(t+1)e^{-t} + C$$

$$= \lim_{b\to\infty}\int_0^b te^{-t} = \lim_{b\to\infty}\left[\frac{-(t+1)}{e^t}\right]_0^b = \left(\lim_{b\to\infty}\frac{-(b+1)}{e^b}\right) - (-1)$$

(l'Hôpital) $\displaystyle = \lim_{b\to\infty}\left(\frac{-1}{e^b}\right) + 1 = 0 + 1 = 1$

5. (12 points) Determine a value for each of the following or, if they do not have values, then show that they diverge or do not exist.

(a) $\displaystyle\lim_{n\to\infty} \frac{n^2 - 2n^3}{n^2 + 3n^3} \frac{/n^3}{/n^3} = \lim_{n\to\infty} \frac{1/n - 2}{1/n + 3} = -\frac{2}{3}$

(b) $\displaystyle\lim_{n\to\infty} \frac{\ln(n)}{n^2} = \text{“}\frac{\infty}{\infty}\text{”}$ (l'Hôpital) $\displaystyle = \lim_{n\to\infty} \frac{1/n}{2n}$

$\displaystyle = \lim_{n\to\infty} \frac{1}{2n^2} = 0$

(c) $\displaystyle\sum_{n=0}^{\infty} \left(\frac{1}{5}\right)^n$ (Geometric Series Formula)

$\displaystyle = \frac{1}{1 - 1/5} = \frac{1}{4/5} = 5/4$

(d) $\displaystyle\sum_{n=0}^{\infty} \frac{2^n + 3^n}{4^n} = \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n + \sum_{n=0}^{\infty} \left(\frac{3}{4}\right)^n$

$\displaystyle = \frac{1}{1 - 1/2} + \frac{1}{1 - 3/4} = 2 + 4 = 6$

6. (10 points)

(a) Find $\int_1^5 \frac{60}{x}\,dx$. $= 60\ln|x|\Big|_1^5 = 60\ln 5$

(b) Approximate the integral from (a) using the Trapezoid Rule with 4 equal subintervals.

x_k	y_k
1	60
2	30
3	20
4	15
5	12

$$\boxed{\Delta x = \frac{5-1}{4} = 1}$$

$$\frac{\Delta x}{2}\left(y_0 + 2y_1 + 2y_2 + 2y_3 + y_4\right)$$

$$= \frac{1}{2}\left(60 + 2(30) + 2(20) + 2(15) + 12\right)$$

$$= 101$$

(c) Approximate the integral from (a) using Simpson's Rule with 4 equal subintervals.

$$\frac{\Delta x}{3}\left(y_0 + 4y_1 + 2y_2 + 4y_3 + y_4\right)$$

$$= \frac{1}{3}\left(60 + 4(30) + 2(20) + 4(15) + 12\right)$$

$$= \frac{292}{3} = 97.\overline{3}$$

The End