University of South Carolina

Midterm Examination 1 September 20, 2018

Math 142–H01

Closed book examination Time: 75 minutes

Name Solutions

Instructions:

No notes, books, or calculators are allowed. If you need more space than is provided use the back of the previous page and clearly indicate you have done so. Simplify your final answers. Full credit may not be awarded for insufficient accompanying work.

1		16
2		9
3		9
4		8
5		8
6		10
Total		60

1. (16 points) Find the following integrals.

(a) $\int 2x^4 + 3x^3 - 2x + 4\ dx$

$\frac{2}{5}x^5 + \frac{3}{4}x^4 - x^2 + 4x + C$

(b) $\int \frac{1}{x^2} + e^x + 3^x + \sqrt[3]{x}\ dx$

$-\frac{1}{x} + e^x + \frac{3^x}{\ln 3} + \frac{3}{4}x^{4/3} + C$

(c) $\int \sin(\theta) + \cos(\theta) + \tan(\theta) + \sec(\theta)\ d\theta$

$-\cos\theta + \sin\theta + \ln|\sec\theta| + \ln|\sec\theta + \tan\theta| + C$

(d) $\int \sec^2(t) + \sec(t)\tan(t) + \frac{1}{1+t^2} + \frac{1}{\sqrt{1-t^2}}\ dt$

$\tan t + \sec t + \tan^{-1} t + \sin^{-1} t + C$

2. (9 points) Find the following integrals.

(a) $\int 2xe^{x^2}\,dx$

$$\boxed{\begin{aligned} u &= x^2 \\ du &= 2x\,dx \end{aligned}}$$

$$= \int e^u\,du = e^u + C = e^{x^2} + C$$

(b) $\int 2te^{2t}\,dt$

$$\boxed{\begin{aligned} u &= 2t & v &= \tfrac{1}{2}e^{2t} \\ du &= 2\,dt & dv &= e^{2t}\,dt \end{aligned}}$$

$$= (2t)\left(\tfrac{1}{2}e^{2t}\right) - \int \tfrac{1}{2}e^{2t}\,2\,dt$$

$$= te^{2t} - \tfrac{1}{2}e^{2t} + C$$

(c) $\int \sin^6(\theta)\cos(\theta)\,d\theta$

$$\boxed{\begin{aligned} u &= \sin\theta \\ du &= \cos\theta\,d\theta \end{aligned}}$$

$$= \int u^6\,du = \frac{1}{7}u^7 + C = \frac{1}{7}\sin^7\theta + C$$

3. (9 points) Find the following integrals.

(a) $\displaystyle\int \frac{3x}{4x^2+9}\,dx$

$$\boxed{\begin{array}{l} u = 4x^2+9 \\ du = 8x\,dx \end{array}}$$

$$= \int \frac{3}{8}\,\frac{1}{u}\,du = \frac{3}{8}\ln|u| + C$$

$$= \frac{3}{8}\ln|4x^2+9| + C$$

(b) $\displaystyle\int \frac{3}{4x^2+9}\,dx = \frac{3}{4}\int \frac{1}{x^2+(3/2)^2}\,dx = \frac{3}{4}\left(\frac{2}{3}\right)\tan^{-1}\left(\frac{2}{3}x\right) + C$

$$= \frac{1}{2}\tan^{-1}\left(\frac{2x}{3}\right) + C$$

(c) $\displaystyle\int t^2\cos(2t)\,dt$

$$\boxed{\begin{array}{ll} u = t^2 & v = \frac{1}{2}\sin(2t) \\ du = 2t & dv = \cos(2t)\,dt \end{array}}$$

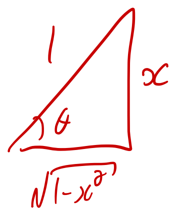
$$= \frac{1}{2}t^2\sin(2t) - \int t\sin 2t\,dt$$

$$\boxed{\begin{array}{ll} u = t & v = -\cos 2t \\ du = dt & dv = \sin 2t\,dt \end{array}}$$

$$= \frac{1}{2}t^2\sin(2t) - \left[-t\cos 2t - \int -\cos 2t\,dt\right]$$

$$= \frac{1}{2}t^2\sin(2t) + t\cos 2t - \frac{1}{2}\sin(2t) + C$$

4. (8 points) Find $\int \frac{dx}{(1-x^2)^{\frac{3}{2}}}$ for $|x| < 1$.



$$\boxed{\begin{aligned} x &= \sin\theta \\ dx &= \cos\theta \, d\theta \\ \sqrt{1-x^2} &= \cos\theta \end{aligned}}$$

$$\int \frac{\cos\theta \, d\theta}{\cos^3\theta} = \int \sec^2\theta \, d\theta = \tan\theta + C$$

$$= \frac{x}{\sqrt{1-x^2}} + C$$

5. (8 points) Find the following integrals.

(a) $\int \sin^5(r)\, dr$

$$= \int (1-\cos^2 r)^2 \sin(r)\, dr \qquad \boxed{\begin{array}{l} u = \cos r \\ du = -\sin r\, dr \end{array}}$$

$$= -\int (1-u^2)^2\, du$$

$$= \int -u^4 + 2u^2 - 1\, du = -\frac{u^5}{5} + \frac{2}{3}u^3 - u + C$$

$$= -\frac{\cos^5(r)}{5} + \frac{2}{3}\cos^3(r) - \cos(r) + C$$

(b) $\int x^3\sqrt{x^2+1}\, dx = \int \frac{1}{2}(x^2)\sqrt{x^2+1}\, 2x\, dx$

$$\boxed{\begin{array}{l} u = x^2+1 \\ du = 2x \\ x^2 = u-1 \end{array}}$$

$$= \frac{1}{2}\int (u-1)\sqrt{u}\, du = \int \frac{1}{2}u^{3/2} - \frac{u^{1/2}}{2}\, du$$

$$= \frac{1}{5}u^{5/2} - \frac{1}{3}u^{3/2} + C$$

$$= \frac{1}{5}(x^2+1)^{5/2} - \frac{1}{3}(x^2+1)^{3/2} + C$$

6. (10 points) Find the following integrals.

(a) $\int \frac{5x-7}{x^2-3x+2}\,dx$

$$\frac{5x-7}{(x-1)(x-2)} = \frac{A}{x-1} + \frac{B}{x-2} = \frac{A(x-2)+B(x-1)}{(x-1)(x-2)}$$

$$5x-7 = (A+B)x + (-2A-B)$$

$A+B=5 \qquad -A=-2 \Rightarrow \boxed{A=2 \quad B=3}$

$-2A-B=-7$

$$= \int \frac{2}{x-1} + \frac{3}{x-2}\,dx = 2\ln|x-1| + 3\ln|x-2| + C$$

(b) $\int \tan^4(\theta)\,d\theta$ $= \int (\sec^2\theta - 1)\tan^2\theta\,d\theta = \int \sec^2\theta\tan^2\theta - \tan^2\theta\,d\theta$

$$= \int \sec^2\theta\tan^2\theta - \sec^2\theta + 1\,d\theta = \int \sec^2\theta\tan^2\theta - \sec^2\theta\,d\theta + \theta$$

$\boxed{\begin{aligned} u &= \tan\theta \\ du &= \sec\theta \end{aligned}}$

$$= \int u^2 - 1\,du + \theta + C$$

$$= \frac{1}{3}u^3 - u + \theta + C$$

$$= \frac{1}{3}\tan^3\theta - \tan\theta + \theta + C$$

The End