University of South Carolina

Midterm Examination 1 September 15, 2015

Math 142 Section H03

Closed book examination Time: 75 minutes

Name Solutions

Instructions:

No notes, books, or calculators are allowed. If you need more space than is provided use the back of the previous page and clearly indicate you have done so. Simplify your final answers. Full credit may not be awarded for insufficient accompanying work.

1	16	16
2	9	9
3	9	9
4	8	8
5	10	10
6	8	8
Total	60	60

1. (16 points) Find the following integrals.

(a) $\displaystyle\int 3x^3 + 2x^2 + 4x - 5\ dx$

$= \frac{3}{4}x^4 + \frac{2}{3}x^3 + 2x^2 - 5x + C$

(b) $\displaystyle\int \frac{1}{r} + e^r + 2^r + \sqrt{r}\ dr$

$= \ln|r| + e^r + \frac{2^r}{\ln(2)} + \frac{2}{3}r^{3/2} + C$

(b) $\displaystyle\int \sin(\theta) + 2\cos(\theta) + 3\tan(\theta) + 4\sec(\theta)\ d\theta$

$-\cos(\theta) + 2\sin\theta + 3\ln|\sec\theta| + 4\ln|\sec\theta + \tan\theta| + C$

(c) $\displaystyle\int \sec^2(x) + \sec(x)\tan(x) + \frac{1}{1+x^2} + \frac{1}{\sqrt{1-x^2}}\ dx$

$= \tan(x) + \sec(x) + \tan^{-1}(x) + \sin^{-1}(x) + C$

2. (9 points) Find the following integrals.

(a) $\int x^2(x^3+2)^6\,dx$

$$\boxed{\begin{aligned} u &= x^3+2 \\ du &= 3x^2\,dx \\ x^2\,dx &= \frac{du}{3} \end{aligned}}$$

$$\Rightarrow \frac{1}{3}\int u^6\,du = \frac{1}{3}\frac{1}{7}u^7 + C$$

$$= \frac{1}{21}(x^3+2)^7 + C$$

(b) $\int \frac{t^2+2t-1}{t-2}\,dt$

$$\boxed{\begin{array}{r|l} & t+4 \\ \hline t-2 & t^2+2t-1 \\ & t^2-2t \\ \hline & 4t-1 \\ & 4t-8 \\ \hline & 7 \end{array}}$$

$$= \int t+4+\frac{7}{t-2}\,dt$$

$$= \frac{1}{2}t^2+4t+7\ln|t-2|+C$$

(c) $\int x\sin(x)\,dx$

$$\boxed{\begin{array}{ll} u = x & v = -\cos(x) \\ du = dx & dv = \sin(x)\,dx \end{array}}$$

$$= x(-\cos x) - \int -\cos(x)\,dx$$

$$= -x\cos x + \sin(x) + C$$

3. (9 points) Find the following integrals.

(a) $\displaystyle\int_1^e x\ln(x)\,dx$ $= \left.(\ln(x))\left(\frac{1}{2}x^2\right)\right|_1^e - \displaystyle\int_1^e \left(\frac{1}{2}x^2\right)\frac{1}{x}\,dx$

$$\boxed{\begin{array}{ll} u=\ln(x) & v=\frac{1}{2}x^2 \\ du=\frac{1}{x}dx & dv = x\,dx \end{array}}$$

$$= \frac{1}{2}e^2 - \frac{1}{2}\int_1^e x\,dx$$

$$= \frac{1}{2}e^2 - \frac{1}{2}\left[\frac{1}{2}x^2\right]_1^e = \frac{1}{4}e^2 + \frac{1}{4}$$

(b) $\displaystyle\int \sin^2(2\theta)\,d\theta$ $= \displaystyle\int \frac{1-\cos(4\theta)}{2}\,d\theta$

$$\boxed{\text{Recall } \sin^2 A = \left(\frac{1-\cos 2A}{2}\right)}$$

$$= \frac{1}{2}\theta - \frac{1}{8}\sin(4\theta) + C$$

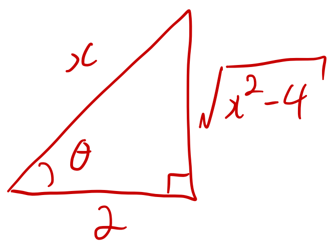
(c) $\displaystyle\int \cos^3(a)\,da$ $= \displaystyle\int (1-\sin^2 a)\cos a\,da$

$$\boxed{\begin{array}{l} u=\sin a \\ du=\cos a\,da \end{array}}$$

$$= \int 1-u^2\,du = u - \frac{u^3}{3} + C$$

$$= \sin(a) - \frac{\sin^3(a)}{3} + C$$

4. (8 points) Find $\int \frac{1}{\left(\sqrt{x^2-4}\right)^3}\, dx$ for $x > 2$.



$$\boxed{\begin{array}{ll} \frac{x}{2} = \sec\theta & \frac{\sqrt{x^2-4}}{2} = \tan\theta \\ x = 2\sec\theta & \\ dx = 2\sec\theta\tan\theta & \sqrt{x^2-4} = 2\tan\theta \end{array}}$$

$$\boxed{0 < \theta < \frac{\pi}{2}}$$

$$= \int \frac{2\sec\theta\tan\theta\, d\theta}{(2\tan\theta)^3} = \frac{1}{4}\int \frac{\sec\theta}{\tan^2\theta}\, d\theta$$

$$= \frac{1}{4}\int \frac{1}{\cos\theta}\, \frac{\cos^2\theta}{\sin^2\theta}\, d\theta = \frac{1}{4}\int \frac{\cos\theta}{\sin^2\theta}\, d\theta$$

$$\boxed{\begin{array}{l} u = \sin\theta \\ du = \cos\theta \end{array}} \quad = \frac{1}{4}\int u^{-2}\, du = -\frac{1}{4}u^{-1} + C$$

$$\boxed{\sin\theta = \frac{\sqrt{x^2-4}}{x}} \quad = -\frac{1}{4}\,\frac{1}{\sin\theta} + C$$

$$= -\frac{x}{4\sqrt{x^2-4}} + C$$

5. (10 points) Find the following integrals.

(a) $\displaystyle\int \frac{2x+7}{x^2+x-2}\,dx$ $= \displaystyle\int \frac{2x+7}{(x-1)(x+2)}\,dx = \int \frac{A}{x-1} + \frac{B}{x+2}\,dx$

$$\boxed{\begin{aligned} &2x+7 = A(x+2) + B(x-1) \\ &\left.\begin{aligned} 2 &= A+B \\ 7 &= 2A-B \end{aligned}\right] \rightarrow A = 3 \quad B = -1 \end{aligned}}$$

$$= \int \frac{3}{x-1} - \frac{1}{x+2}\,dx = 3\ln|x-1| - \ln|x+2|$$

(b) $\displaystyle\int \frac{x+1}{x^2-4x+13}\,dx$ $= \displaystyle\int \frac{x+1}{(x-2)^2+9}\,dx$

$$\boxed{\begin{aligned} u &= x-2 \qquad x = u+2 \\ du &= dx \end{aligned}}$$

$$= \int \frac{(u+2)+1}{u^2+9}\,du = \int \frac{u}{u^2+9}\,du + \int \frac{3}{u^2+9}\,du$$

$$= \frac{1}{2}\ln|u^2+9| + \tan^{-1}\left(\frac{u}{3}\right) + C$$

$$= \frac{1}{2}\ln|x^2-4x+13| + \tan^{-1}\left(\frac{x-2}{3}\right) + C$$

6. (8 points) Determine whether the following integrals converge or diverge. If they converge, find their values.

(a) $\int_0^\infty \frac{1}{4+x^2}\,dx = \lim_{b\to\infty}\int_0^b \frac{1}{4+x^2}\,dx$

$$= \lim_{b\to\infty}\left[\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)\right]_0^b$$

$$= \lim_{b\to\infty} \frac{1}{2}\tan^{-1}\left(\frac{b}{2}\right) - \frac{1}{2}\tan^{-1}(0)$$

$$= \frac{1}{2}\left(\frac{\pi}{2}\right) - \frac{1}{2}(0) = \frac{\pi}{4}$$

Converges to $\pi/4$.

(b) $\int_{-2}^1 \frac{1}{x^2}\,dx = \lim_{b\to 0^-}\int_{-2}^b \frac{1}{x^2}\,dx + \lim_{a\to 0^+}\int_a^1 \frac{1}{x^2}\,dx$

$$= \lim_{b\to 0^-}\left[-\frac{1}{x}\right]_{-2}^b + \lim_{a\to 0^+}\left[-\frac{1}{x}\right]_a^1$$

$$= \cancel{\lim_{b\to 0^-}\left(-\frac{1}{b}\right)}^{-\infty} - \left(-\frac{1}{-2}\right) + \left(-\frac{1}{1}\right) - \cancel{\lim_{a\to 0^+}\left(-\frac{1}{a}\right)}^{-\infty}$$

DNE

$\Rightarrow$ Diverges.

The End