University of South Carolina

Final Examination   December 9, 2021

Math 142–005/006

Closed book examination   Time: 150 minutes

Name _____

Instructions:

Notes, books, computer, phones, calculators or other aids are **not** allowed. Please write on only one side of each page. If you need more space than is provided, then ask for extra paper from the proctor. Simplify your final answers. Full credit will not be awarded for insufficient accompanying work.

There are
$$16 + 9 + 10 + 8 + 12 + 8 + 9 + 8 + 8 + 10 + 10 + 8 = 116$$

points available, but the exam is **out of** 100.
(In other words, there are 16 bonus points available)

1. (16 points) Find the following integrals.

(a) $\int 2x^3 - 3x^2 - 4x + 2 \ dx$

(b) $\int e^x + 2^x + \sqrt[2]{x} + \dfrac{1}{x} \ dx$

(c) $\int \cos(\theta) + \sin(\theta) + \tan(\theta) + \sec(\theta) \ d\theta$

(d) $\int \sec(x)\tan(x) + \sec^2(x) + \dfrac{1}{1+x^2} + \dfrac{1}{\sqrt{1-x^2}} \ dx$

2. (9 points) Find the following integrals.

(a) $\displaystyle\int 2x\cos(x^2)\ dx$

(b) $\displaystyle\int \cos^2(5\theta)\ d\theta$

(c) $\displaystyle\int x^2\cos 2x\ dx$

3. (10 points) Find the following integrals.

(a) $\displaystyle\int \frac{2x+3}{x^2-4x+4}\,dx$

(b) $\displaystyle\int_0^\infty \frac{1}{4x^2+9}\,dx$

4. (8 points) Find $\displaystyle\int \frac{1}{\sqrt{4x^2 - 49}}\, dx$ for $x > \frac{7}{2}$.

5. (12 points) For each of the following functions:

- write down the Maclaurin series using Σ notation, and

- write down the radius of convergence.

(You do not need to justify your answers.)

(a) e^x

(b) $\cos(x)$

(c) $\sqrt{1+x}$

(d) $\ln(1+x)$

6. (8 points) Determine the following limits:

(a) $\lim\limits_{n\to\infty} \dfrac{n^2 - 2 - n^3}{5n^3 - 4n^2 - 1}$.

(b) $\lim\limits_{n\to\infty} \dfrac{2^{2n+1} - 3^n}{4^n + 3^n}$.

(c) $\lim\limits_{n\to\infty} 3\,(2n)^{\frac{2}{n}}$.

(d) $\lim\limits_{x\to 0} \dfrac{\cos(3x) - 1}{x\sin(x)}$.

7. (9 points) For each of the following series, determine if it converges or diverges.

(a) $\displaystyle\sum_{n=3}^{\infty} \frac{n+1}{2n^3 - 1}$.

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{3^n}{n^2}$.

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{(n+1)!}{(2n)!}$.

8. (8 points) Determine the Taylor polynomial of order 3 generated by the function $\tan(x)$ at $x = \pi$.

9. (8 points) Determine the interval of convergence for the power series

$$\sum_{n=1}^{\infty} \frac{(2x+5)^n}{6n} \quad .$$

10. (10 points) Find the area of the region enclosed by $r = 1 + \sin(\theta)$.

11. (10 points)

(a) Find Cartesian coordinates for each of the following points in polar coordinates:

- $(3, 0)$

- $(1, 5\pi/6)$

- $(-1, \pi/2)$

(b) Find polar coordinates for each of the following points in Cartesian coordinates:

- $(-1, -1)$

- $(0, 1)$

- $(2\sqrt{3}, 2)$

(c) Find a polar equation equivalent to the Cartesian equation $x^2 = 5y$.

(d) Find a Cartesian equation equivalent to the polar equation $r = \csc(\theta)$.

12. (8 points) Determine the length of the polar curve given by $r = \theta^2 - 1$ from $\theta = \pi$ to $\theta = 2\pi$.

The End