University of South Carolina

Final Examination December 7, 2021

Math 142–003/004

Closed book examination | Time: 150 minutes

Name ____________________________________

Instructions:

Notes, books, computer, phones, calculators or other aids are **not** allowed. Please write on only one side of each page. If you need more space than is provided, then ask for extra paper from the proctor. Simplify your final answers. Full credit will not be awarded for insufficient accompanying work.

There are

$$16 + 9 + 10 + 8 + 12 + 8 + 9 + 8 + 8 + 10 + 10 + 8 = 116$$

points available, but the exam is **out of** 100.
(In other words, there are 16 bonus points available)

1. (16 points) Find the following integrals.

(a) $\displaystyle\int 4x^4 - x^3 + 2x - 1 \; dx$

(b) $\displaystyle\int e^x + 2^x + \sqrt[2]{x} + \frac{1}{x} \; dx$

(c) $\displaystyle\int \cos(\theta) + \sin(\theta) + \tan(\theta) + \sec(\theta) \; d\theta$

(d) $\displaystyle\int \sec(x)\tan(x) + \sec^2(x) + \frac{1}{1+x^2} + \frac{1}{\sqrt{1-x^2}} \; dx$

2. (9 points) Find the following integrals.

(a) $\int 2xe^{x^2}\,dx$

(b) $\int \sin^2(3\theta)\,d\theta$

(c) $\int x^2e^{2x}\,dx$

3. (10 points) Find the following integrals.

(a) $\displaystyle\int \frac{2x+3}{x^2-2x+1}\,dx$

(b) $\displaystyle\int_0^2 \frac{x}{x^2-1}\,dx$

4. (8 points) Find $\int \frac{1}{\sqrt{4x^2 - 49}}\, dx$ for $x > \frac{7}{2}$.

5. (12 points) For each of the following functions:

- write down the Maclaurin series using Σ notation, and
- write down the radius of convergence.

(You do not need to justify your answers.)

(a) e^x

(b) $\sin(x)$

(c) $\sqrt{1+x}$

(d) $\ln(1+x)$

6. (8 points) Determine the following limits:

(a) $\lim_{n\to\infty} \dfrac{3n^2 - 2 + n}{5n - 4n^2 - 1}$.

(b) $\lim_{n\to\infty} \dfrac{4^n + 3^n}{2^{2n+1} - 3^n}$.

(c) $\lim_{n\to\infty} n \sin\left(\dfrac{2}{n}\right)$.

(d) $\lim_{x\to 0} \dfrac{\cos(2x) - 1}{\ln(1 + x) - x}$.

7. (9 points) For each of the following series, determine if it converges or diverges.

(a) $\displaystyle\sum_{n=3}^{\infty} \frac{n-1}{2n^2+1}$.

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{n^3}$.

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{3^n}{(2n)!}$.

8. (8 points) Determine the Taylor polynomial of order 3 generated by the function $\tan(x)$ at $x = \pi$.

9. (8 points) Determine the interval of convergence for the power series

$$\sum_{n=1}^{\infty} \frac{(3x-2)^n}{4n}\ .$$

10. (10 points)

(a) Using the Taylor polynomial of order 3 generated by the function $f(x) = \sin(x)$ at $x = 0$, estimate the value of $\sin(0.1)$.

(b) Recall that the remainder R_n of the Taylor polynomial of f at a is bounded by

$$|R_n(x)| \leq \frac{M}{(n+1)!}|x-a|^{n+1}$$

where $|f^{(n+1)}(t)| \leq M$ for all t between x and a. Find an upper bound on the absolute value of the error for the estimate from (a) using Taylor's remainder theorem with $M = 1$.

(c) What order of Taylor polynomial will ensure that the estimate for $\sin(0.1)$ is within 10^{-9} of the actual value?

11. (10 points)

(a) Find Cartesian coordinates for each of the following points in polar coordinates:

- $(2, 0)$
- $(2, \pi/6)$
- $(-1, \pi/2)$

(b) Find polar coordinates for each of the following points in Cartesian coordinates:

- $(-1, 1)$
- $(0, -1)$
- $(2\sqrt{3}, 2)$

(c) Find a polar equation equivalent to the Cartesian equation $y^2 = 4x$.

(d) Find a Cartesian equation equivalent to the polar equation $r = \sec(\theta)$.

12. (8 points) Consider the curve C described by the parametric equations

$$x = t(t^2 - 3)$$
$$y = 3(t^2 - 3).$$

Find the Cartesian coordinates of all points on C at which the tangent line is either horizontal or vertical.

The End