University of South Carolina

Final Examination December 8, 2020

Math 142–011/012/031/032

Closed book examination Time: 150 minutes

Name _____

Instructions:

No notes, books, computer, phones, calculators or other aids are allowed. You must be alone while you are taking the test and you should not be in contact electronically or physically with any other person from the time you start the exam until you submit your final solutions. Do not use more time than the time allotted. Full credit will not be awarded for insufficient accompanying work.

Submit your completed exam on Blackboard by uploading a scanned PDF file or as multiple JPEG image files. You do not have to print the exam, but please have a separate page/image for each page of the exam. Clearly indicate which questions you are answering on every image/page.

1		9
2		9
3		10
4		8
5		8
6		9
7		9
8		8
9		8
10		10
11		10
12		8
Total		106

1. (9 points) Find the following integrals.

(a) $\int \theta^2 \sin(\theta^3)\ d\theta$

(b) $\int 4t\cos(3t)\ dt$

(c) $\int \sin^2(2\theta)\ d\theta$

2. (9 points) Find the following integrals.

(a) $\int x^2 e^x \, dx$

(b) $\int \frac{7}{9x^2 + 1} \, dx$

(c) $\int \sin^2(\theta) \cos^2(\theta) \, d\theta$

3. (10 points) Find the following integrals.

(a) $\displaystyle\int \frac{3x-5}{2x^2-7x+6}\,dx$

(b) $\displaystyle\int_1^{\infty} \frac{1}{s^2+2s+1}\,ds$

4. (8 points) Find $\displaystyle\int \frac{dr}{(4-r^2)^{\frac{3}{2}}}$ for $|r| < 2$.

5. (8 points) Determine the following limits:

(a) $\lim_{n\to\infty} \dfrac{n^2 - 2^n}{3n^2 + 2^n}$.

(b) $\lim_{n\to\infty} \dfrac{2n^3 - 2}{4^n}$.

(c) $\lim_{n\to\infty} (1+n)^{1/n}$.

(d) $\lim_{x\to 0} \dfrac{1 - e^{x^3}}{x - \sin(x)}$.

6. (9 points) For each of the following series, determine if it converges or diverges.

(a) $\sum_{n=1}^{\infty} \frac{3}{n^2}$.

(b) $\sum_{n=4}^{\infty} \frac{\ln(n)^2}{n}$.

(c) $\sum_{n=0}^{\infty} \frac{4n^2+2}{3n^3-1}$.

7. (9 points) For each of the following series, determine if it converges or diverges.

(a) $\displaystyle\sum_{n=3}^{\infty} \frac{(2n+1)!}{(n!)^2}$.

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-2)^n}{4^n}$.

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{n! + 5n}{4^{-n}}$.

8. (8 points) Determine the Taylor polynomial of order 3 generated by the function $\sec(x)$ at $x = \pi$.

9. (8 points) Determine the interval of convergence for the power series

$$\sum_{n=1}^{\infty} \frac{(6x-5)^n}{n^3} \ .$$

10. (6 points)

(a) Using the Maclaurin polynomial of order 4 for $f(x) = e^x$, estimate the value of $\dfrac{1}{e^2}$.

(b) What is the maximum value of $|f^{(5)}(x)|$ on the interval $[-2, 0]$?

(c) Find an upper bound on the absolute value of the error for the estimate from (a) using the Remainder Estimation Theorem.

11. (10 points)

(a) Find Cartesian coordinates for each of the following points in polar coordinates:

- $(2, -\pi)$
- $(3, \pi/2)$
- $(2, \pi/6)$

(b) Find polar coordinates for each of the following points in Cartesian coordinates:

- $(2, 2)$
- $(-1, 0)$
- $(\sqrt{3}, -1)$

(c) Find a polar equation equivalent to the Cartesian equation $4xy = x^2 + y^2$.

(d) Find a Cartesian equation equivalent to the polar equation $\cos(\theta) - \sin(\theta) = 4r$.

12. (8 points) Let C be the parametric curve determined by

$$x = 4t^2 - 1$$
$$y = t^2 - 2t$$

where t is a real parameter.

(a) Determine the x and y coordinates of the point when $t = 2$.

(b) Determine $\left.\dfrac{dy}{dx}\right|_{t=2}$.

(c) Find an equation for the line tangent to the curve C at the point where $t = 2$.

(d) Determine $\left.\dfrac{d^2y}{dx^2}\right|_{t=2}$.

The End