University of South Carolina

Final Examination December 11, 2018

Math 142 Section H01

Closed book examination Time: 150 minutes

Name ______________________________

Instructions:

No notes, books, or calculators are allowed. If you need more space than is provided, then ask the proctor for additional paper. Be sure to write your name on every page. Simplify your final answers. **Full credit may not be awarded for insufficient accompanying work.**

1		16
2		9
3		10
4		8
5		12
6		8
7		9
8		8
9		8
10		10
11		10
12		8
Total		116

1. (16 points) Find the following integrals.

(a) $\displaystyle\int 3x^3 - 4x^2 - 9x + 4 \; dx$

(b) $\displaystyle\int e^x + 2^x + \sqrt[3]{x} + \ln(x) \; dx$

(c) $\displaystyle\int \cos(\theta) + \sin(\theta) + \tan(\theta) + \sec(\theta) \; d\theta$

(d) $\displaystyle\int \sec(x)\tan(x) + \sec^2(x) + \frac{1}{1+x^2} + \frac{1}{\sqrt{1-x^2}} \; dx$

2. (9 points) Find the following integrals.

(a) $\int 2x \sin(x^2)\ dx$

(b) $\int \sin^2(2\theta)\ d\theta$

(c) $\int x \cos(x)\ dx$

3. (10 points) Find the following integrals.

(a) $\displaystyle\int \frac{5x-7}{x^2-3x+2}\,dx$

(b) $\displaystyle\int \frac{1}{e^x-1}\,dx$

4. (8 points) Find $\displaystyle\int \frac{4}{\left(\sqrt{4x^2-1}\right)^3}\,dx$ for $x > \frac{1}{2}$.

5. (12 points) For each of the following functions:

- write down the Maclaurin series using Σ notation, and
- write down the radius of convergence.

(You do not need to justify your answers.)

(a) e^x

(b) $\cos(x)$

(c) $(1+x)^{\frac{1}{3}}$

(d) $\tan^{-1}(x)$

6. (8 points) Determine the following limits:

(a) $\lim_{n\to\infty} \dfrac{2n^2 - 18}{4n^2 - 4n + 1}$.

(b) $\lim_{n\to\infty} \dfrac{2(3^n) + n^2}{n^3 - 3^n}$.

(c) $\lim_{n\to\infty} 7(2n)^{3/n}$.

(d) $\lim_{x\to 0} \dfrac{2e^x - 2 - 2x - x^2}{\sin(x) - x}$.

7. (9 points) For each of the following series, determine if it converges or diverges.

(a) $\displaystyle\sum_{n=3}^{\infty} \frac{n^3 - n + 1}{2n^2 - n + 1}$.

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{3^n}{n^4}$.

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{4^n}{(n-1)!}$.

8. (8 points) Determine the Taylor polynomial of order 3 generated by the function $\sec(x)$ at $x = \pi$.

9. (8 points) Determine the interval of convergence for the power series

$$\sum_{n=1}^{\infty} \frac{(2x-1)^n}{n} .$$

10. (10 points)

(a) Using the Taylor polynomial of order 3 generated by the function $f(x) = e^x$ at $x = 0$, estimate the value of $e^{-0.1}$.

(b) Find an upper bound on the absolute value of the error for the estimate from (a) using Taylor's remainder theorem.

(c) What order of Taylor polynomial will ensure that the estimate for $e^{-0.1}$ is within 10^{-10} of the actual value?

11. (10 points)

(a) Find Cartesian coordinates for each of the following points in polar coordinates:

- $(1, 1)$
- $(2, \pi/3)$
- $(-1, \pi)$

(b) Find polar coordinates for each of the following points in Cartesian coordinates:

- $(1, -1)$
- $(0, 1)$
- $(2, 2\sqrt{3})$

(c) Find a polar equation equivalent to the Cartesian equation $4y^2 - 2x + 1 = 0$.

(d) Find a Cartesian equation equivalent to the polar equation $r\tan(\theta) = 1$.

12. (8 points)
Determine the length of the curve obtained from the graph of the function

$$f(x) = \frac{e^x + e^{-x}}{2}$$

from $x = -1$ to $x = 1$.

The End