

University of South Carolina
Final Examination December 14, 2017
Math 142 Section 005/006

Closed book examination

Time: 150 minutes

Name _____

Instructions:

No notes, books, or calculators are allowed. If you need more space than is provided, then ask the proctor for additional paper. Be sure to write your name on every page. Simplify your final answers. **Full credit may not be awarded for insufficient accompanying work.**

1		16
2		9
3		10
4		8
5		12
6		8
7		9
8		8
9		8
10		10
11		10
12		8
Total		116

1. (16 points) Find the following integrals.

(a) $\int 3x^5 - 4x^2 - 5x + 2 \, dx$

(b) $\int e^x + 2^x + \sqrt{x} + \ln(x) \, dx$

(c) $\int \cos(\theta) + \sin(\theta) + \tan(\theta) + \sec(\theta) \, d\theta$

(d) $\int \sec(x) \tan(x) + \sec^2(x) + \frac{1}{1+x^2} + \frac{1}{\sqrt{1-x^2}} \, dx$

2. (9 points) Find the following integrals.

(a) $\int \sin(e^x)e^x dx$

(b) $\int \theta \cos(\theta) d\theta$

(c) $\int \sin^2(\theta) d\theta$

3. (10 points) Find the following integrals.

(a) $\int \frac{5x + 1}{x^2 + x - 2} dx$

(b) $\int \frac{2s + 1}{s^2 + 4} ds$

4. (8 points) Find $\int \sqrt{1 - 9r^2} \, dr$.

5. (12 points) For each of the following functions:

- write down the Maclaurin series using Σ notation, and
- write down the radius of convergence.

(You do not need to justify your answers.)

(a) e^x

(b) $\sin(x)$

(c) $\ln(x + 1)$

(d) $\frac{1}{1+x}$

6. (8 points) Determine the following limits:

(a) $\lim_{n \rightarrow \infty} \frac{2n^2 - 4}{3n + 1}$.

(b) $\lim_{n \rightarrow \infty} \frac{n \ln(n)}{n^2 + 1}$.

(c) $\lim_{n \rightarrow \infty} (3n)^{2/n}$.

(d) $\lim_{x \rightarrow 0} \frac{\sin(x) - x}{x^2 \ln(x + 1)}$.

7. (9 points) For each of the following series, determine if it converges or diverges.

(a) $\sum_{n=1}^{\infty} \frac{n!}{n^2}$.

(b) $\sum_{n=0}^{\infty} \frac{3n}{2^n}$.

(c) $\sum_{n=0}^{\infty} \frac{n^2 - 2}{n + 3}$.

8. (8 points) Determine the Taylor polynomial of order 3 generated by the function $\sec(x)$ at $x = 0$.

9. (8 points) Determine the interval of convergence for the power series

$$\sum_{n=1}^{\infty} \frac{(3x - 4)^n}{n} .$$

10. (10 points)

(a) Estimate the integral $\int_0^2 x^3 dx$ using the Trapezoid rule with 4 equal subintervals.

(b) Let E_T be the error of the Trapezoid rule applied to the integral $\int_a^b f(x) dx$ with n equal subintervals. Recall that $|E_T| \leq \frac{M(b-a)^3}{12n^2}$ where M is an upper bound for the values of $|f^{(2)}(x)|$ on $[a, b]$. What is a bound on the error in the estimate from part (a)?

(c) What is the actual value of the integral in part (a)?

11. (10 points)

(a) Find Cartesian coordinates for each of the following points in polar coordinates:

- $(5, \pi)$

- $(2, -\pi)$

- $(1, \pi/6)$

(b) Find polar coordinates for each of the following points in Cartesian coordinates:

- $(-3, 3)$

- $(0, -1)$

- $(4\sqrt{3}, 4)$

(c) Find a polar equation equivalent to the Cartesian equation $x^2 + y^2 = xy - 4$.

(d) Find a Cartesian equation equivalent to the polar equation $\cos^2(\theta) = 1 - 2\sin^2(\theta)$.

12. (8 points)

Determine the length of the parametric curve given by

$$\begin{aligned}x &= t^3 - 3t \\ y &= 3t^2\end{aligned}$$

where $1 \leq t \leq 3$.