**University of South Carolina**

Final Examination December 12, 2017

**Math 142 Section 003/004**

Closed book examination Time: 150 minutes

**Name** ______________________________

**Instructions:**

No notes, books, or calculators are allowed. If you need more space than is provided, then ask the proctor for additional paper. Be sure to write your name on every page. Simplify your final answers. **Full credit may not be awarded for insufficient accompanying work.**

| | | |
|---|---|---|
| 1 | | 16 |
| 2 | | 9 |
| 3 | | 10 |
| 4 | | 8 |
| 5 | | 12 |
| 6 | | 8 |
| 7 | | 9 |
| 8 | | 8 |
| 9 | | 8 |
| 10 | | 10 |
| 11 | | 10 |
| 12 | | 8 |
| Total | | 116 |

1. (16 points) Find the following integrals.

(a) $\displaystyle\int 3x^5 - 4x^2 - 5x + 2 \; dx$

(b) $\displaystyle\int e^x + 2^x + \sqrt{x} + \ln(x) \; dx$

(c) $\displaystyle\int \cos(\theta) + \sin(\theta) + \tan(\theta) + \sec(\theta) \; d\theta$

(d) $\displaystyle\int \sec(x)\tan(x) + \sec^2(x) + \frac{1}{1+x^2} + \frac{1}{\sqrt{1-x^2}} \; dx$

2. (9 points) Find the following integrals.

(a) $\int 2xe^{-x^2}\ dx$

(b) $\int x\sin(x)\ dx$

(c) $\int \cos^2(\theta)\ d\theta$

3. (10 points) Find the following integrals.

(a) $\displaystyle\int \frac{3x-7}{x^2-5x+6}\,dx$

(b) $\displaystyle\int \frac{3x^2+29}{x^2+9}\,dx$

4. (8 points) Find $\int \sqrt{25 - t^2}\, dt$.

5. (12 points) For each of the following functions:

- write down the Maclaurin series using $\Sigma$ notation, and
- write down the radius of convergence.

(You do not need to justify your answers.)

(a) $e^x$

(b) $\sin(x)$

(c) $(1+x)^{\frac{1}{2}}$

(d) $\ln(x+1)$

6. (8 points) Determine the following limits:

(a) $\lim_{n\to\infty} \frac{3n^2 - n}{4n^3 - 2n + 5}$.

(b) $\lim_{n\to\infty} \frac{3^n}{n^3}$.

(c) $\lim_{n\to\infty} 7n^{2/n}$.

(d) $\lim_{x\to 0} \frac{x^2 e^x}{\cos(x) - 1}$.

7. (9 points) For each of the following series, determine if it converges or diverges.

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{n^3}{4^n}$.

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{3^n}{(n+1)!}$.

(c) $\displaystyle\sum_{n=0}^{\infty} \frac{4n+1}{n^3+3}$.

8. (8 points) Determine the Taylor polynomial of order 3 generated by the function $\tan(x)$ at $x = 0$.

9. (8 points) Determine the interval of convergence for the power series

$$\sum_{n=1}^{\infty} \frac{(3x-2)^n}{n^2} .$$

10. (10 points)

(a) Estimate the integral $\int_2^6 \frac{1}{x-1}\,dx$ using the Trapezoid rule with 4 equal subintervals.

(b) Let $E_T$ be the error of the Trapezoid rule applied to the integral $\int_a^b f(x)\,dx$ with $n$ equal subintervals. Recall that $|E_T| \leq \frac{M(b-a)^3}{12n^2}$ where $M$ is an upper bound for the values of $|f^{(2)}(x)|$ on $[a,b]$. What is a bound on the error in the estimate from part (a)?

(c) What is the actual value of the integral in part (a)?

11. (10 points)

(a) Find Cartesian coordinates for each of the following points in polar coordinates:

- $(2, 0)$
- $(1, \pi)$
- $(4, \pi/4)$

(b) Find polar coordinates for each of the following points in Cartesian coordinates:

- $(2, -2)$
- $(0, 2)$
- $(1, \sqrt{3})$

(c) Find a polar equation equivalent to the Cartesian equation $x^2 - y^2 = 4$.

(d) Find a Cartesian equation equivalent to the polar equation $r\sin(\theta) = 4 - r^2$.

12. (8 points)
Determine the length of the polar curve given by $r = \theta^2 - 1$ from $\theta = \pi$ to $\theta = 2\pi$.

**The End**