University of South Carolina

Final Examination December 8, 2016

Math 142 Section H03

Closed book examination Time: 150 minutes

Name ______________________________

Instructions:

No notes, books, or calculators are allowed. If you need more space than is provided use the back of the previous page and clearly indicate you have done so. Simplify your final answers. **Full credit may not be awarded for insufficient accompanying work.**

1		16
2		9
3		10
4		8
5		8
6		8
7		9
8		8
9		8
10		10
11		10
12		8
Total		112

1. (16 points) Find the following integrals.

(a) $\displaystyle\int 2x^4 - 3x^3 + 2x + 1 \; dx$

(b) $\displaystyle\int e^x + 3^x + \sqrt[3]{x} - \frac{1}{x^2} \; dx$

(b) $\displaystyle\int \cos(\theta) + \sin(\theta) + \tan(\theta) + \sec(\theta) \; d\theta$

(c) $\displaystyle\int \sec(x)\tan(x) + \sec^2(x) + \frac{1}{4 + x^2} + \frac{1}{\sqrt{1 - x^2}} \; dx$

2. (9 points) Find the following integrals.

(a) $\int x^2 \sin(x)\ dx$

(b) $\int 3t(t^2 - 7)^4\ dt$

(c) $\int \cos^2(3\theta)\ d\theta$

3. (10 points) Find the following integrals.

(a) $\displaystyle\int \frac{2x+7}{x^2+5x+6}\,dx$

(b) $\displaystyle\int_0^2 \frac{4s}{16+s^4}\,ds$

4. (8 points) Find $\int \frac{dr}{r^2\sqrt{r^2-9}}$ for $r > 3$.

5. (8 points)

(a) Write down the Maclaurin series for e^x in Σ notation.

(b) Write down the Maclaurin series for $\ln(1+x)$ in Σ notation.

(c) Determine the value of $\displaystyle\sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^n$ or explain why the series diverges.

(d) Determine the value of $\displaystyle\sum_{n=0}^{\infty}\frac{2^n+1}{3^n}$ or explain why the series diverges.

6. (8 points) Determine the following limits:

(a) $\lim_{n\to\infty} \frac{n^2 - 1}{2n^2 + 3n}$.

(b) $\lim_{n\to\infty} \frac{\ln(n)}{n}$.

(c) $\lim_{n\to\infty} (4n)^{2/n}$.

(d) $\lim_{x\to 0} \frac{x - \sin(x)}{x^2 \sin(x)}$.

7. (9 points) For each of the following series, determine if it converges or diverges.

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{2^n}{n!}$.

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{n-2}{n^2+4}$.

(c) $\displaystyle\sum_{n=0}^{\infty} \frac{2^n}{n^2}$.

8. (8 points) Determine the Taylor polynomial of order 4 generated by the function $e^x \cos(x)$ at $x = \pi$.

9. (8 points) Determine the interval of convergence for the power series

$$\sum_{n=1}^{\infty} \frac{(x-1)^n}{n^3 3^n}\ .$$

10. (10 points)

(a) Using the Taylor polynomial of order 3 generated by the function $f(x) = \sin(x)$ at $x = 0$, estimate the value of $\sin(0.1)$.

(b) Find an upper bound on the absolute value of the error for the estimate from (a) using the remainder estimation theorem.

(c) What order of Taylor polynomial will ensure that the estimate for $\sin(0.1)$ is within 10^{-9}?

11. (10 points)

(a) Find Cartesian coordinates for each of the following points in polar coordinates:

- $(2, 0)$
- $(3, \pi)$
- $(2, \pi/3)$

(b) Find polar coordinates for each of the following points in Cartesian coordinates:

- $(0, 2)$
- $(-1, -1)$
- $(\frac{1}{2}, \frac{\sqrt{3}}{2})$

(c) Find a polar equation equivalent to the Cartesian equation $x^2 - 2 = y$.

(d) Find a Cartesian equation equivalent to the polar equation $r^2 + 1 = r\cos(\theta)$.

12. (8 points)
Determine the length of the parametric curve given by $x = 2t^3$, $y = 3t^2$ where $0 \le t \le \sqrt{8}$.

The End