SAMPLE

University of South Carolina

Sample Midterm Examination 3   April 7, 2022

Math 142–001/002

Closed book examination   Time: 75 minutes

Name ⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯

Instructions:

Notes, books, computer, phones, calculators or other aids are **not** allowed. Please write on only one side of each page. If you need more space than is provided, then ask for extra paper from the proctor. Simplify your final answers. Full credit will not be awarded for insufficient accompanying work.

There are $12 + 10 + 12 + 8 + 8 + 8 = 58$ points available, but the exam is **out of** 55.
(In other words, there are 3 bonus points available)

1. (12 points) For each of the following functions:

   - write down the Maclaurin series using Σ notation, and

   - write down the radius of convergence.

(You do not need to justify your answers.)

(a) e^x

(b) $\cos(x)$

(c) $(1+x)^{\frac{1}{3}}$

(d) $\tan^{-1}(x)$

2. (10 points) Determine the interval of convergence for the power series

$$\sum_{n=3}^{\infty} \frac{(6x-2)^n}{4n+3}.$$

3. (12 points) The power series

$$f(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \cdots$$

$$g(x) = \sum_{n=0}^{\infty} nx^n = x + 2x^2 + 3x^3 + 4x^4 + 5x^5 + \cdots$$

converge for $-1 < x < 1$ (you don't need to show this). Find the first 4 non-zero terms of the following power series:

(a) $f(x) + g(x)$

(b) $f(2x^2)$

(c) $g'(x)$

(d) $\int_0^x g(y)\, dy$

4. (8 points) Determine the Taylor polynomial of order 3 generated by the function $\sec(x)$ at $x = \pi$.

5. (8 points) Find the following:

(a) $\displaystyle\lim_{x \to 0} \frac{\tan^{-1}(x) - x}{\sin(x) - x}$

(b) $\displaystyle\sum_{n=0}^{\infty} \left(\frac{1}{3}\right)^n$

(c) $\displaystyle\sum_{n=0}^{\infty} \frac{4^n}{n!}$

6. (8 points) Use the Taylor polynomial of order 3 generated by $\sin(x)$ at $x = 0$ to estimate

$$\int_0^3 \frac{\sin(2x)}{x}\, dx \ .$$

The End