

University of South Carolina

Midterm Examination 2 March 17, 2022

Math 142–001/002

Closed book examination

Time: 75 minutes

Name _____

Instructions:

Notes, books, computer, phones, calculators or other aids are **not** allowed. Please write on only one side of each page. If you need more space than is provided, then ask for extra paper from the proctor. Simplify your final answers. Full credit will not be awarded for insufficient accompanying work.

There are $9 + 9 + 9 + 9 + 12 + 8 = 56$ points available, but the exam is **out of 50**.
(In other words, there are 6 bonus points available)

1. (9 points) Find the limit of each of the following sequences or explain why the limit does not exist.

(a) $\lim_{n \rightarrow \infty} \frac{n^3 - 5n}{2n^3 + 4n}$

(b) $\lim_{n \rightarrow \infty} \left(\frac{1}{n}\right)^n$

(c) $\lim_{n \rightarrow \infty} \frac{2 - 3^n}{5 - 3^n}$

2. (9 points) Find the value of each of the following series or explain why the series diverges.

(a) $\sum_{n=1}^{\infty} n$

(b) $\sum_{n=0}^{\infty} \frac{4 - 2^n}{3^n}$

(c) $\sum_{n=2}^{\infty} \left(\frac{1}{7}\right)^n$

3. (9 points) For each series, what can you conclude from the given convergence test?

(a) $\sum_{n=0}^{\infty} \frac{n!}{(2n)!}$ using the Ratio Test.

(b) $\sum_{n=0}^{\infty} 2^{-n}$ using the Integral Test.

(c) $\sum_{n=1}^{\infty} \frac{4^n}{n^3}$ using the Root Test.

4. (9 points) For each series, what can you conclude from the given convergence test?

(a) $\sum_{n=1}^{\infty} \frac{4n}{n^2 + 2}$ using the Limit Comparison Test with $\sum \frac{1}{n^2}$.

(b) $\sum_{n=4}^{\infty} \frac{1}{n^3 + 1}$ using the Limit Comparison Test with $\sum \frac{1}{n^3}$.

(c) $\sum_{n=2}^{\infty} \frac{1}{n^2 + 1}$ using the Direct Comparison Test with $\sum \frac{1}{n^2}$.

5. (12 points) For each of the following series, determine if it converges or diverges.

$$(a) \sum_{n=0}^{\infty} \frac{3^n}{(2n+1)!}$$

$$(b) \sum_{n=2}^{\infty} \frac{4n^2 + 1}{n^3 - 1}$$

$$(c) \sum_{n=0}^{\infty} \frac{2^n}{(n+5)^n}$$

6. (8 points) For each of the following series, determine if it

- converges absolutely,
- converges conditionally, or
- diverges.

(a) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$

(b) $\sum_{n=2}^{\infty} \frac{(-1)^n}{n!}$