SAMPLE

University of South Carolina

Final Examination May 3, 2022

Math 142–001/002

Closed book examination | Time: 150 minutes

Name ________________________________

Instructions:

Notes, books, computer, phones, calculators or other aids are **not** allowed. Please write on only one side of each page. If you need more space than is provided, then ask for extra paper from the proctor. Simplify your final answers. Full credit will not be awarded for insufficient accompanying work.

There are

$$16 + 9 + 10 + 8 + 12 + 8 + 9 + 8 + 8 + 10 + 10 + 8 = 116$$

points available, but the exam is **out of** 110.
(In other words, there are 6 bonus points available)

1. (16 points) Find the following integrals.

(a) $\int 3x^3 - 4x^2 - 9x + 4 \; dx$

(b) $\int e^x + 2^x + \sqrt[3]{x} + \ln(x) \; dx$

(c) $\int \cos(\theta) + \sin(\theta) + \tan(\theta) + \sec(\theta) \; d\theta$

(d) $\int \sec(x)\tan(x) + \sec^2(x) + \frac{1}{1+x^2} + \frac{1}{\sqrt{1-x^2}} \; dx$

2. (9 points) Find the following integrals.

(a) $\int 2x \sin(x^2)\ dx$

(b) $\int \sin^2(2\theta)\ d\theta$

(c) $\int x \cos(x)\ dx$

3. (10 points) Find the following integrals.

(a) $\displaystyle\int \frac{5x-7}{x^2-3x+2}\,dx$

(b) $\displaystyle\int \frac{1}{e^x-1}\,dx$

4. (8 points) Find $\displaystyle\int \frac{4}{\left(\sqrt{4x^2-1}\right)^3}\,dx$ for $x > \frac{1}{2}$.

5. (12 points) For each of the following functions:

- write down the Maclaurin series using Σ notation, and
- write down the radius of convergence.

(You do not need to justify your answers.)

(a) e^x

(b) $\cos(x)$

(c) $(1+x)^{\frac{1}{3}}$

(d) $\tan^{-1}(x)$

6. (8 points) Determine the following limits:

(a) $\lim_{n\to\infty} \frac{2n^2 - 18}{4n^2 - 4n + 1}$.

(b) $\lim_{n\to\infty} \frac{2(3^n) + n^2}{n^3 - 3^n}$.

(c) $\lim_{n\to\infty} 7(2n)^{3/n}$.

(d) $\lim_{x\to 0} \frac{2e^x - 2 - 2x - x^2}{\sin(x) - x}$.

7. (9 points) For each of the following series, determine if it converges or diverges.

(a) $\displaystyle\sum_{n=3}^{\infty} \frac{n^3 - n + 1}{2n^2 - n + 1}$.

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{3^n}{n^4}$.

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{4^n}{(n-1)!}$.

8. (8 points) Determine the Taylor polynomial of order 3 generated by the function $\sec(x)$ at $x = \pi$.

9. (8 points) Determine the interval of convergence for the power series

$$\sum_{n=1}^{\infty} \frac{(2x-1)^n}{n} .$$

10. (10 points) Find the area of the region enclosed by $r = 1 + \cos(\theta)$.

11. (10 points)

(a) Find Cartesian coordinates for each of the following points in polar coordinates:

- $(1, 1)$
- $(2, \pi/3)$
- $(-1, \pi)$

(b) Find polar coordinates for each of the following points in Cartesian coordinates:

- $(1, -1)$
- $(0, 1)$
- $(2, 2\sqrt{3})$

(c) Find a polar equation equivalent to the Cartesian equation $4y^2 - 2x + 1 = 0$.

(d) Find a Cartesian equation equivalent to the polar equation $r\tan(\theta) = 1$.

12. (8 points)
Determine the length of the curve obtained from the graph of the function

$$f(x) = \frac{e^x + e^{-x}}{2}$$

from $x = -1$ to $x = 1$.

The End