You have 50 minutes to complete the exam.

Problem 1 Determine whether each of the following statements are true or false. No justification is necessary.

1. If G is a finite group with subgroup H, then the number of left cosets is equal to the number of right cosets.
2. If two abelian groups have the same order, then they are isomorphic.
3. If $\varphi : G \to H$ is a group homomorphism, then $\ker(\varphi)$ is a normal subgroup of G.
4. Every subgroup of an abelian group is a normal subgroup.
5. A finite group is never isomorphic to an infinite group.

Problem 2 Let $G = S_4 \times S_3$ and let $H = A_4 \times A_3$ be a normal subgroup. Determine the order of $((1234), (123))H$ in G/H.

Problem 3 Determine the number of isomorphism classes of finite abelian groups of order 64.

Problem 4 Let $\phi : G \to H$ be a group homomorphism and assume H is abelian. Show that if $x, y \in G$, then $xyx^{-1}y^{-1} \in \ker(\varphi)$.

Problem 5 Prove that S_4 and $A_4 \times \mathbb{Z}_2$ are not isomorphic.