You have 50 minutes to complete the exam.

Problem 1 Determine whether each of the following statements are true or false. No justification is necessary.

1. If G is finite group with normal subgroup N, then $|G| = |G/N||N|$.
2. If G is a finite group and $x \in G$, then $o(x)$ divides $|G|$.
3. If $\varphi : G \to H$ is a group homomorphism, then $\mathrm{im}(\varphi)$ is a normal subgroup of H.
4. Every cyclic group is abelian.
5. If a group is not abelian, then it has no normal subgroups.

Problem 2 Let $G = \mathbb{Z}_{12} \times \mathbb{Z}_{12}$ and let $H = \langle (6,4) \rangle$ be a normal subgroup. Determine the order of the element $x = (9,10) + H$ in G/H.

Problem 3 Determine all isomorphism classes of finite abelian groups of order 180.

Problem 4 Let G be a group and consider the function $f : G \to G$ given by $f(g) = g^{-1}$. Prove that G is abelian if and only if f is a homomorphism.

Problem 5 Show that if $G/Z(G)$ is cyclic, then G is abelian.