

You have 50 minutes to complete the exam.

Problem 1 Determine whether each of the following statements are true or false. No justification is necessary.

1. If G is a group with normal subgroup H , then $G \cong H \times (G/H)$.
2. If two cyclic groups have the same order, then they are isomorphic.
3. Let G, H, K be groups. If $G \cong H$ and $H \cong K$, then $G \cong K$.
4. Every group of prime order is cyclic.
5. Every homomorphism of abelian groups is an isomorphism.

Problem 2 Let $G = \mathbb{Z}_{12} \times \mathbb{Z}_{12}$ and let $H = \langle (6, 4) \rangle$ be a normal subgroup. Determine the order of the group G/H .

Problem 3 Determine all isomorphism classes of finite abelian groups of order 200.

Problem 4 Let $\varphi : \mathbb{Z}_8 \rightarrow \mathbb{Z}_8$ be the function given by $\varphi(x) = 5x$. Prove that φ is an isomorphism.

Problem 5 Let D_4 be the dihedral group of order 8. Prove that $Z(D_4) \cong \mathbb{Z}_2$.