You have 50 minutes to complete the exam.

Problem 1 Determine whether each of the following statements are true or false. No justification is necessary.

1. If H is a subgroup of a finite group G, then the order of H divides the order of G.
2. Suppose G is a finite group of order n and $x \in G$. Then $x^n = e$.
3. Every homomorphism of abelian groups is an isomorphism.
4. If G and H are subgroups of the same order, then $G \cong H$.
5. If $\varphi : G \to H$ is a group homomorphism, then $\operatorname{im}(\varphi)$ is a subgroup of H.

Problem 2 Let $G = \mathbb{Z}_{30} \times \mathbb{Z}_{10}$ and let $H = \langle (25, 5) \rangle$ be a normal subgroup. Determine the order of the group G/H.

Problem 3 Determine the isomorphism classes of finite abelian groups of order 72.

Problem 4 Let $\varphi : \mathbb{R} \to \mathrm{GL}_2(\mathbb{R})$ the function given by

$$\varphi(a) := \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix}.$$

Prove that φ is a group homomorphism.

Problem 5 Let $\varphi : G \to H$ be a group homomorphism. Prove that φ is injective if and only if $\ker(\varphi) = \{e\}$.