

You have 50 minutes to complete the exam.

Problem 1 Determine whether each of the following statements are true or false. No justification is necessary.

1. Every subgroup of a cyclic group is abelian.
2. Every subgroup of an abelian group is cyclic.
3. Every permutation can be written as a product of disjoint cycles.
4. The symmetric group on X is the set of functions $f : X \rightarrow X$.
5. If every *proper* subgroup of a group G is cyclic, then G itself is cyclic.

Problem 2 List all elements of $\mathbb{Z}_6 \times \mathbb{Z}_{15}$ of order 3.

Problem 3 Determine the order of the element $(1346)(234)(15)(12)$ in the group S_6 .

Problem 4 Let H be a subgroup of a group G . Let a be an element of G and suppose K is the set of all elements of the form axa^{-1} for some $x \in G$. Prove that K is a subgroup of G .

Problem 5 Let H be the subgroup $H = \{e, (23)\}$ in the symmetric group S_3 . Describe all the distinct right cosets of H in S_3 .