You have 50 minutes to complete the exam.

Problem 1 Determine whether each of the following statements are true or false. No justification is necessary.

1. If g, h are elements of a group G, then $o(g)o(h) = o(gh)$.
2. If H is a subgroup of a group G and K is a subgroup of H, then K is a subgroup of G.
3. A direct product of abelian groups is abelian.
4. Every permutation can be written *uniquely* as a product of transpositions.
5. An r-cycle is even if and only if r is even.

Problem 2 List all subgroups of the group $\mathbb{Z}_{30}$.

Problem 3 Rewrite the permutation

$$(135)(245)(67)(243)^{-1}(12)(15)$$

as a product of disjoint cycles.

Problem 4 Let G be the subset of the symmetric group S_6 such that $f(4) = 4$ for all $f \in G$. Prove that G is a subgroup of S_6.

Problem 5 Let G be a group. For $a, b \in G$, write $a \sim b$ if there exists $x \in G$ such that $a = xbx^{-1}$. Prove that $\sim$ is an equivalence relation on G.