You have 50 minutes to complete the exam.

Problem 1 Determine whether each of the following statements are true or false. No justification is necessary.

1. Every cyclic group is abelian.
2. If every *proper* subgroup of a group G is cyclic, then G itself is cyclic.
3. If H is a subgroup of G, then H contains the identity of G.
4. If σ is an element of S_n, then the order of σ divides n.
5. Suppose $\sim$ is an equivalence relation on a set X. For $x, y \in X$, the equivalence classes $\overline{x}$ and $\overline{y}$ are equal if and only if $x \sim y$.

Problem 2 Determine the order of the element $(9, 81)$ in the group $\mathbb{Z}_{24} \times \mathbb{Z}_{90}$.

Problem 3 Rewrite the permutation

$$f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 4 & 6 & 1 & 9 & 7 & 8 & 3 & 5 \end{pmatrix}$$

as a product of transpositions. Is f even or odd?

Problem 4 Let G be an abelian group and suppose H is the subset of G where $x \in H$ if and only if $x^3 = e$. Prove that H is a subgroup of G.

Problem 5 Suppose that f is an r-cycle in S_n where r is odd. Prove that f^2 is also an r-cycle.