You have 50 minutes to complete the exam.

Problem 1 Determine whether each of the following statements are true or false. No justification is necessary.

1. An element x of a group G has order n if and only if $x^n = e$.

2. If H and K are subgroups of a group G, then $H \cap K$ is a subgroup.

3. If G, H are groups, then $|G \times H| = |G| \times |H|$.

4. A product of odd permutations is odd.

5. Let H be a subgroup of G. Every right coset of H in G is also a subgroup of G.

Problem 2 Determine the order of $(6, 70)$ in $\mathbb{Z}_{60} \times \mathbb{Z}_{80}$.

Problem 3 Rewrite the permutation

$$(1234)(56)(142)^{-1}(14)(15)$$

as a product of disjoint cycles.

Problem 4 Let g be an element of a group G. Let H be the subset of G such that $hg = gh$ for all $h \in H$. Prove that H is a subgroup of G.

Problem 5 Prove that if σ is a transposition in S_n then there exists a permutation $\tau \in S_n$ such that $\tau \circ \sigma \circ \tau^{-1} = (1\ 2)$.