You have 50 minutes to complete the exam.

Problem 1 Indicate whether the following statements are true or false. You do not need to justify your answers and no partial credit will be awarded.

1. If $(G, *)$ is a group, then G must be a finite set.
2. $(\mathbb{Q}, +)$ is a group.
3. If $*$ is an associative binary operation, then it is commutative.
4. If x has finite order, then $x^n = e$ for every $n \in \mathbb{Z}$.
5. The set of all $n \times n$ matrices with real entries forms a group under matrix multiplication.

Problem 2 Find $\gcd(302, 58)$ and two integers x, y such that $302x + 58y = \gcd(302, 58)$.

Problem 3 Let G be the set of positive rational numbers and consider the binary operation $x * y = \frac{1}{x} + \frac{1}{y}$ on G. Is $*$ associative? Is $*$ commutative? Does $(G, *)$ form a group?

Problem 4 Let G be a group. Prove that G is abelian if and only if $(xy)^{-1} = x^{-1}y^{-1}$ for all $x, y \in G$.

Problem 5 Prove that, if n and m are relatively prime positive integers with $m \neq 1$, then $\mathbb{Z}_n$ has no elements of order m.