You have 50 minutes to complete the exam.

Problem 1 Indicate whether the following statements are true or false. You do not need to justify your answers and no partial credit will be awarded.

1. If $(G, *)$ is a group, then $*$ is commutative.
2. Division is a binary operation on $\mathbb{Q}$.
3. Let $*$ be an associative binary operation on a set G. If $x * y = x * z$, then $y = z$.
4. The set $\mathbb{Z}_n$ contains exactly n elements.
5. The set of all $n \times m$ matrices with real entries forms a group under matrix addition.

Problem 2 Find $\gcd(104, 46)$ and two integers x, y such that $104x + 46y = \gcd(104, 46)$.

Problem 3 Let G be the set of all 2×2 matrices of the form $\begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix}$, where $a \in \mathbb{R}$. Prove that G forms a group under matrix multiplication.

Problem 4 Let G be a set with a commutative binary operation $*$. Suppose that for all $a, b, c \in G$, the equation $a * (b * c) = b * (c * a)$ holds. Prove that $*$ is associative.

Problem 5 Let G be an abelian group and let $x, y \in G$. Prove that $o(xy)$ divides $o(x)o(y)$.