You have 50 minutes to complete the exam.

Problem 1 Indicate whether the following statements are true or false. You do not need to justify your answers and no partial credit will be awarded.

1. Every binary operation is associative.

2. If x, y are elements of a group G, and $x = y^{-1}$, then $y = x^{-1}$.

3. The set of integers $\mathbb{Z}$ form a group under multiplication.

4. Subtraction is an associative binary operation on $\mathbb{Z}$.

5. If x is an element in a group G of order 1, then x is the identity.

Problem 2 Find $\gcd(72, 56)$ and two integers x, y such that $72x + 56y = \gcd(72, 56)$.

Problem 3 Let G be the set of positive integers and consider the binary operation $x * y = 2x + 2y$ on G. Is $*$ associative? Is $*$ commutative? Does $(G, *)$ form a group?

Problem 4 Let G be a group. Suppose $xyz = yzx$ for all $x, y, z \in G$. Prove that G is abelian.

Problem 5 Let n and d be positive integers. Let M be a matrix of order n in the general linear group $\mathrm{GL}(d, \mathbb{R})$. Let M^T denote the transpose matrix of M. Show that M^T is in $\mathrm{GL}(d, \mathbb{R})$ and M^T has order n.