University of South Carolina
MATH 544-001

Midterm Examination 3 - Sample C

November 14, 2024

Closed book examination Time: 75 minutes

Instructions:

Notes, books, computer, phones, calculators or other aids are **not** allowed. Please write on only one side of each page. If you need more space than is provided, then ask for extra paper from the proctor. Simplify your final answers. Full credit will not be awarded for insufficient accompanying work.

There are 30 total points available.

Problem 1. (5 points) Find a basis for the null space of the following matrix:

$$\begin{pmatrix} 1 & 1 & 5 & 0 & -3 \\ 1 & 2 & 8 & 0 & -7 \\ -2 & 0 & -4 & 1 & 0 \\ 2 & -1 & 1 & 0 & 6 \end{pmatrix}$$

Problem 2. (5 points) Find a basis for the subspace of $\mathbb{R}^3$ spanned by

$$\left\{ \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 2 \\ 4 \\ 6 \end{pmatrix}, \begin{pmatrix} 1 \\ 4 \\ 5 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \right\}.$$

Problem 3. (5 points) Compute the change-of-basis matrix $\underset{\mathcal{C}\leftarrow\mathcal{B}}{P}$ where

$$\mathcal{B} = \left\{ \begin{pmatrix} 3 \\ 2 \\ 9 \end{pmatrix}, \begin{pmatrix} 8 \\ 12 \\ 8 \end{pmatrix}, \begin{pmatrix} 10 \\ 2 \\ 6 \end{pmatrix} \right\} \text{ and } \mathcal{C} = \left\{ \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix} \right\}$$

are bases of $\mathbb{R}^3$.

Determine whether each of the following statements are true or false. No justification is necessary.

Problem 4. (1 point) A subset H of a vector space V is a subspace of V if and only if the zero vector is in H.

Problem 5. (1 point) The set of polynomials of degree exactly 2 form a subspace of all polynomials.

Problem 6. (1 point) The column space of an $m \times n$ matrix is $\mathbb{R}^m$.

Problem 7. (1 point) The kernel of a linear transformation is a vector space.

Problem 8. (1 point) Let $\mathcal{B}$ be a set of vectors in a vector space V. If $V = \text{span}(\mathcal{B})$ and $\mathcal{B}$ is linearly independent, then $\mathcal{B}$ is a basis for V.

Problem 9. (5 points) Let $S = \{p_1, \ldots, p_n\}$ be a set of polynomials such that no two have the same degree. Prove that S is linearly independent.

Problem 10. (5 points) Suppose that W_1 and W_2 are subspaces of a finite-dimensional vector space V. Prove that

$$\dim(W_1 \cap W_2) \leq \min\left(\dim(W_1), \dim(W_2)\right).$$

The End