University of South Carolina

MATH 544-001

Midterm Examination 3

November 14, 2024

Closed book examination Time: 75 minutes

Instructions:

Notes, books, computer, phones, calculators or other aids are **not** allowed. Please write on only one side of each page. If you need more space than is provided, then ask for extra paper from the proctor. Simplify your final answers. Full credit will not be awarded for insufficient accompanying work.

There are 30 total points available.

Problem 1. (5 points) Find a basis for the null space of the following matrix:

$$\begin{pmatrix} 1 & 0 & 2 & 3 & 4 & 5 \\ 2 & 4 & 4 & 6 & 0 & 10 \\ 4 & 4 & 8 & 0 & 1 & 2 \end{pmatrix}$$

Problem 2. (5 points) Find a basis for the column space of the following matrix:

$$\begin{pmatrix} 4 & 2 & 2 & 6 & 8 & 4 \\ 2 & 0 & 1 & 3 & 0 & 2 \\ 0 & 1 & 0 & 0 & 4 & 0 \\ 1 & 1 & 2 & 6 & 0 & 1 \end{pmatrix}$$

Problem 3. (5 points) The set

$$\mathcal{B} = \left\{ \begin{pmatrix} 1 \\ 2 \\ 3 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ 4 \\ 5 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix} \right\}$$

is a basis for $\mathbb{R}^4$. Find the coordinate vector $[\mathbf{x}]_{\mathcal{B}}$ where

$$\mathbf{x} = \begin{pmatrix} 1 \\ 1 \\ 2 \\ 5 \end{pmatrix}.$$

Determine whether each of the following statements are true or false. No justification is necessary.

Problem 4. (1 point) The dimension of a vector space V is the number of elements in V.

Problem 5. (1 point) If $\mathcal{B}$ is a basis of a vector space V, then span $\mathcal{B} = V$ and $\mathcal{B}$ is linearly independent.

Problem 6. (1 point) If T is an invertible linear transformation, then the kernel and the range are isomorphic vector spaces.

Problem 7. (1 point) If H is a subspace of a finite-dimensional vector space V, then $\dim(H) \leq \dim(V)$.

Problem 8. (1 point) Every vector space has a unique basis.

Problem 9. (5 points) Let $\mathbb{P}_8$ be the set of polynomials of degree at most 8. Let V be the subset of $\mathbb{P}_8$ consisting of polynomials f such that $f(4) = 0$. Prove that V is a subspace of $\mathbb{P}_8$.

Problem 10. (5 points) Let A be an $n \times n$ matrix such that $A^2 = 0$. Prove that the column space $\mathrm{Col}(A)$ is a subset of the null space $\mathrm{Nul}(A)$.

The End