University of South Carolina

MATH 544-001

Midterm Examination 2

October 10, 2024

Closed book examination Time: 75 minutes

Instructions:

Notes, books, computer, phones, calculators or other aids are **not** allowed. Please write on only one side of each page. If you need more space than is provided, then ask for extra paper from the proctor. Simplify your final answers. Full credit will not be awarded for insufficient accompanying work.

There are 30 total points available.

Consider the matrices

$$A = \begin{pmatrix} 1 & 0 & 4 \\ 2 & -1 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 2 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}.$$

For each of the following, compute the matrix or indicate that the expression is undefined.

Problem 1. (2 points) $\det(C)A + CA$

Problem 2. (3 points) $2AB + C^{-1}$

Problem 3. (5 points) Find the inverse of the matrix

$$\begin{pmatrix} 1 & 1 & 0 \\ 1 & 4 & 2 \\ 2 & 10 & 6 \end{pmatrix}$$

or show that it does not exist.

Problem 4. (5 points) Find the determinant of the following matrix:

$$\begin{pmatrix} 1 & 2 & 4 & 0 \\ 1 & 3 & 2 & 0 \\ 5 & 17 & 12 & 1 \\ 4 & 2 & 3 & 0 \end{pmatrix}$$

Determine whether each of the following statements are true or false. No justification is necessary.

Problem 5. (1 point) Let A, B, C be $n \times n$ matrices. Then $A(B+C) = AB + AC$.

Problem 6. (1 point) Every function f can be written $f(x) = Ax$ for some matrix A.

Problem 7. (1 point) Let A be an $n \times n$ matrix. The matrix A is invertible if and only if A^T is invertible.

Problem 8. (1 point) Let A be an $n \times n$ matrix. The matrix A is row equivalent to the $n \times n$ identity matrix if and only if the columns of A span $\mathbb{R}^n$.

Problem 9. (1 point) Let A, B be $n \times n$ matrices. Then $\det(A+B) = \det(A) + \det(B)$.

Problem 10. (5 points) Let A, B, C be $n \times n$ matrices. Suppose C is invertible and $AC = CB$. Prove that A is invertible if and only if B is invertible.

Problem 11. (5 points) Let $T : \mathbb{R}^n \to \mathbb{R}^m$ and $S : \mathbb{R}^m \to \mathbb{R}^p$ be surjective linear transformations. Prove that the composition $S \circ T : \mathbb{R}^n \to \mathbb{R}^p$ is a surjective linear transformation.

The End